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STEREOMETRICAL

# Propoſitions:

VARIOUSLY APPLICABLE;

But



PARTICULARLY INTENDED

For

# GAGEING.



By

*ROBERT ANDERSON.*

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To the
# READER

ABout seven years since I resolved these following propositions, with others of the like nature; but did not intend ever to have published them: But being over-perswaded by some of my Friends; I have here made so many of them publick, as I thought would be convenient to what here is intended.

Here I ought to acknowledge that great Respect which I owe to my Worthy Friend Mr. *John Collins*, one of the *Royal Society*; which propounded to me about 6 years since, that proposition of the second Sections of the *Sphere* and *Spheroide*; and also in giving me Information, and helping me to such Books as might be most advandageous: Further, not only I, but all Lovers of the Mathematicks are very much oblieged to this our Worthy Friend for his good Intelligence, great care in sending for Books from beyond the Seas, and his continual love in promoting the Mathematicks, and Mathematical Men.

Fo

Forasmuch as throughout this little Tract, here is much use made of *Parallelepipedons*, *Prismes* and *Pyramides*: I think fit, a little to insist thereon; chiefly for the young Learners sakes, for which this is only intended.

In the *Diagram*,

Let there be a *Solid*, as P O H G V F I R; the *plane* P O H G parallel equal and alike to the *plane* R I F V; Further, P O I R is parallel equal and like to the *plane* G H F V: Yet further, the *plane* O H F I is parallel equal and like the *plane* P R V G: such a *Solid* is called a *Parallelepipedon*.

Let there be a *Solid* as G H D E V F, the *plane* G E V is parallel equal and like the *plane* H D F; Further, the *planes* H G F V and D E V F are alike and the line F V common to both: such a *Solid* is called a *Prisme*.

Let there be a *Solid* as H B C D F, the *planes* B C F, C D F, D H F and

HBF all meet at the point F, such a *Solid* is called a *Pyramide*.

Further,

*Pyramides*, *Prismes* and *Parallelepipedons* upon the same *Base* and Altitude, are as, one, one and a half, and three, that is, if the *Pyramide* be 4, the *prisme* will be 6, and the *parallelepipedon* will be 12.

*An Example in Numbers.*

If P G, 6; P O, 8; and H F the Altitude be 9; Then multiply 6 by 8, and the product will be 48, the *Area* P O H G, this *Area* multiplyed by 9 the Altitude, the product is 432, the *parallelepipedon* P O H G V F I R, if the *Area* 48 be multiplyed by half 9 that is 4½ the product will be 216, equal to a *prisme* of the *Base* P O H G and altitude H F.

If the *Area* P O H G, 48, be multiplyed by one third of 9, that is by 3, the product will be 144 equal to a *pyramide* whose *Base* is P O H G and altitude H F.

To

*To find the solidity of the prisme GHDEVF.*

Let GE, 10; GH, 8; HF 9; multiply 10 by 8, and the product will be 80 the *Area* GHDE, this 80 being multiplyed by half the altitude HF, 9; that is, 4½, the product will be 360, the *prisme* GHDEVF.

*To find the solidity of the* pyramide *HBCDF.*

Let HD, 10; HB, 4; multiply 10 by 4 the product will be 40, the *Area* HBCD, this *Area* multiplyed by 3, that is one third of the Altitude *HF*, the product will be 120, the *pyramide* HBCDF.

To


*To find the ſolidity of a* Solid, *that hath one of its* Baſes *an* Ellipſis *and the other* Baſe *a Circle*, *theſe two* Baſes *are ſuppoſed to be parallel.*

In ſuch a *Solid*, the ſides being continued will never meet at a point as in circular and *Elliptick cones*, and therefore the general Rule will not reſolve theſe kind of *Solids* : therefore conſider it thus ;

Let

Let P E be equal to the Transverse diameter in an *Ellipsis*, P A its Conjugate diameter. Let R V and R I be the diameters of a Circle. First, Let there be made a Rectangled figure of the Transverse and Conjugate diameters of the *Ellipsis*, as P A C E. Let there be a square made of the diameter R V, as R I F V. Let the altitude of the *Solid* be H F, find the solidity of the *Solid* P A C E V F I R, thus, Let it be cut into as many *parallelepipedons*, *prismes*, & *pyramides* as are necessary, then find the solidity of those *parallelepipedons*, *prismes* and *pyramides*, as before is taught, those *parallelepipedons*, *prismes* & *pyramides* being added together will be equal to the whole *Solid*, thus, the *parallepipedon* P O H G V F I R, more the *prisme* G E D H F V, more the *pyramide* D C B H F, more the *prisme* H B A O I F; these 4 *Solids* being added together make the whole *Solid* A C E P R I F V. For the whole

is

is, 4; the product is 40; equal to the *prisme* GHDEVF. HD, 3; HB, 1; therefore HBCD is equal to 3; which being multiplyed by one third of the altitude HF, that is, 3; the *pyramide* HBCDF will be equal to 9; BA, 3; AO, 1; therefore ABHO will be equal to 3, which being multiplyed by half HF the altitude, that is, by 4; the product is 13 ; equal to the *prisme* OABHFI. Now these 4 composed Numbers being added together make 144 equal to the whole *frustum pyramide* PACEVFIR. Then as 14 : 11 :: 144 : 113; the irregular *Elliptick solid*, whose Transverse PE, and conjugate PA diameters at the *Elliptick Base* are 6 and 4, and the diameters of the Circular *Base* is RV 3.

Here note,

Although *Archimedes* gives the proportion of the square of the diameter of any Circle, to the *area* of that Circle;

Circle ; as 14, is to 11. And alſo ; that the diameter of any Circle is to the circumference of that Circle, as 7, is to 22 ; Yet you are not to underſtand that theſe proportions are in their juſt values; But that they are the leaſt numbers that will beſt agree to that approach.

If thoſe numbers be thought not to agree near enough to truth, inſtead of thoſe, theſe may be uſed ; for, 14 and 11, take 452 and 355 ; for 7 and 22, take 113 and 355.

Further note.

In the 24th. propoſition there is ſomewhat ſaid of *Cylindrick hoofs*. Thoſe *cylinders* ought to be upright and not inclining *cylinders*.

Another note:

By reaſon of much buſineſs, I could not attend the *Preſs* as I ought to have done ; and by that means there are conſiderable faults, which I deſire may be corrected before the Book is read, other-

otherwise there may rise a misapprehension of what there is intended.

Lastly note,

Almost at the beginning of the first proposition you may find it printed, Thus, CN: NZ :: EH: HZ, 4. 6. the meaning is thus, as CN is to NZ; so is EH, to HZ, by the fourth prop. of the sixth of *Euclid*; The like in the rest.

*R. A.*

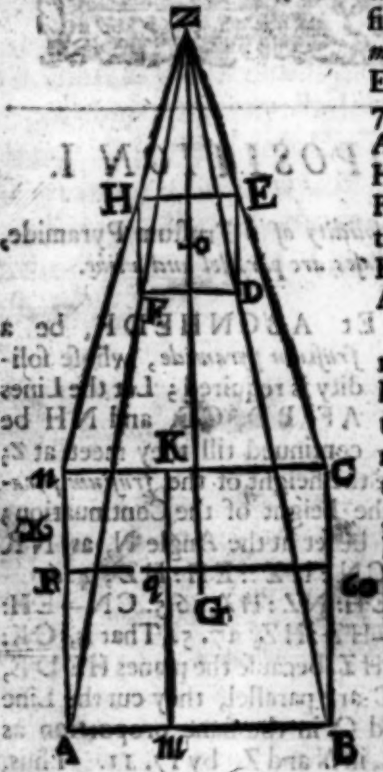
# PROPOSITION I.

*To find the solidity of a* Frustum Pyramide, *whose bases are parallel and a like.*

LEt ABCNHEDF be a *frustum pyramide*, whose solidity is required; Let the Lines AF, BD, CE, and NH be continued till they meet at Z; Let GO, be the height of the *frustum pyramide*, OZ the height of the Continuation; Let FDEH be set at the Angle N, as NRQK. Then CN : NZ :: EH : HZ, 4. 6.

And CN:EH::NZ:HZ, 16. 5. CN—EH: EH :: NZ—HZ : HZ, 17. 5. That is, CK: KN :: NH : HZ, because the planes HFDE, and NABC are parallel, they cut the Line ZG, in G and O in the same proportion as they cut NZ, in N and Z, by 17. 11. Thus, NH : HZ :: GO : OZ, Therefore CK : KN :: GO : OZ, 11. 5. OZ being found, which

added to G O makes the whole Altitude G Z; by which find the whole *pyramide* N A B C Z,




which done, find the *pyramide* H F D E Z, by the 7. 12. then N A B C Z less H F D E Z, the Remainder is the *frustum* H F D E C B A N.

A 2d way may be thus; First to prove that A K is a mean proportion between A C and F E; then C N : N K :: C A : A K, 4. 6. A N : N R :: N M : N Q, 1. 6. C N : N K :: A N : N R, 7. 5. C A : A K :: N M : N Q, 11. 5.

Further,

Further, if G O the Altitude of the *frustum* F D E H C B A N, be multiplied by A C more A K more K R, that Product will be the triple of the said *frustum*; for the parallelepipedon made of G Z the altitude of the *pyramide* in the Base N A B C is triple of the *pyramide* N A B C Z, by the same reason the parallelepipedon made of G O in A C, together with the parallelepipedon made by Z O in A C is triple of the said *pyramide* by 7. 12. if the parallelepipedon made of O Z in F E, that is, in R K, that is, the triple of the *pyramide* H F D E Z be taken from the parallelepipedon N A B C Z, there remains that which is made of G O in A C, together with those that are made of O Z in A Q and the same O Z in M C, the triple of the *frustum* A B C N H E D F, by 5. 5. for, C K : K N :: G O : O Z, C K : K N :: A R : R N, Therefore G O : O Z :: A R : R N, and C K : K N :: C M : M N, 4. 6. A R : R N :: A Q : Q N, therefore G O : O Z :: C M : M N, 11. 5. G O : O Z :: A Q : Q N, therefore G O in M N = O Z in C M, G O in Q N = O Z in A Q, add G O in A C to both, that is, G O in M N, more G O in Q N, more G O in A C, is equal to O Z in C M, more O Z in A Q, more G O in A C; but O Z in C M, more O Z in A Q, more G O in A C are triple of the said *frustum*; therefore G O in M N, more G O in Q N,

 more

more GO in AC are equal to the triple of the said *frustum.*

*An Example in Numbers, and first of the first.*

CK, 34 : KN, 26 :: GO, 68 : OZ, 52; then 68 more 52=120, GZ : then AC 3600; in GZ, 120; is equal to 432000, a third part is 144000, ABCNZ, FE, 676 in OZ, 52 is equal to 35152.

A third part is 11717⅓, HFDEZ, 144000 less 11717⅓ is equal to 132282⅔ the solidity of the *frustum* ACEF.

*The Second way.*

AC, 3600 more FE, 676, more AK, 1560 the sum of those are 5836, which multiplyed by GO, 68, is 396848, a third part is 132282⅔. These ways *Christopher-Clavius* hath, pag. 208 of his *Geometria practica.* I shall give a third in conformity to what follows.

*A Third way.*

Let ACEZHPOF be a *frustum pyramide* the Bases Squares and Parallel; Let HPOF be projected in the Base ACFZ, as ZVRT; VR and TR be continued to D and B; then will TD be equal to RA, and RC a Square; most manifest it is, that the parallelepipedon ZVRTPOFH, more the prisme VABROP, more the *pyramide* RBCDO, more the prisme RDETOF is equal to the said *frustum pyramide.* For

For all the parts of any magnitude being taken, together are equal to the whole. *Axiom* 19. 1.

The prisme VABROP is equal to the prisme RDETFO, because of equal bases and altitude, but the prism VABROP is half the parallelepipedon, made of the base VABR, and the altitude RO, by 28. 11. therefore the parallelepipedon made of the base ZABT, that is the Rectangle made of HF and ZA, and the altitude RO, more the *pyramide* RBDCO that is $\frac{1}{3}$ of the square of the difference of the sides of the upper and lower Bases and the altitude RO, shall be equal to the *frustum* proposed.


*Example in Numbers.*

ZE, 60; ZT, 26; the product 1560 equal to BZ, a third part of RC is 385 $\frac{1}{3}$, which

added to 1560, is equal to 1945 ⅓, this last composed number multiplyed by 68 equal to R O makes 132282 ⅓ the solidity of the said *frustum*; then as 14 to 11, or more near the truth, as 452 to 355, so is 132282 ⅓ to 103894 $\frac{124}{113}$ the *frustum* cone adscribed within the *frustum pyramide*.

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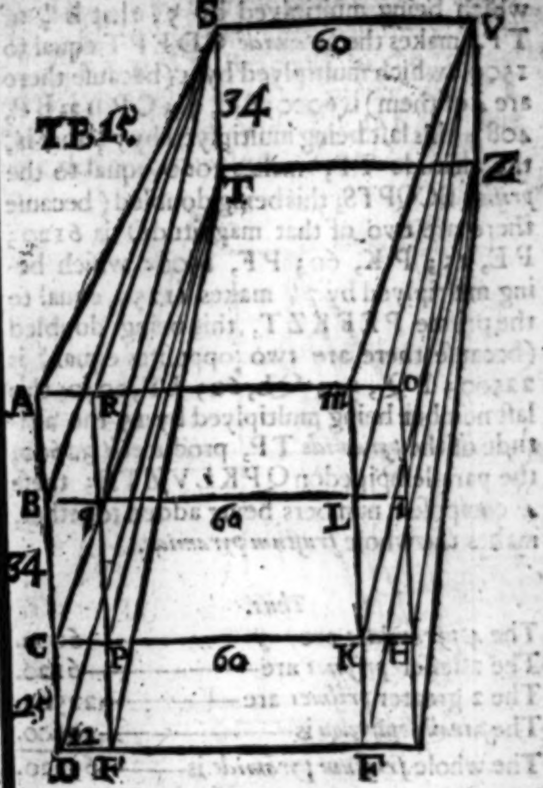
## PROPOSITION II.

*To find the solidity of a* frustum pyramide, *whose Bases are parallel, but not alike; that is, when their corresponding sides are not proportional.*

LET ADGOVZTS be the *pyramide* proposed, here A G is a square, S Z a parallelogram; Let S T Z V be projected in the base, as, Q P K L; continue the sides, as Q L, to B and I; P K, to C and H; L K, to M and F; P Q, to E and R; the Figure being completed, the whole *frustum* will be composed of these parts, namely 4 *pyramides*, as C D E P T, K F G H Z, L I O M V, R Q B A S, more 4 prismes, as B C P Q T S, Q R M L V S, L K H I V Z, K F E P T Z, more the parallelepipedon P Q L K Z V S T; a further demonstration is needless.




*An Example.*

C D equal to 25, D E equal to 12, the like for the other Angles, C E equal to 300,

which being multiplyed by 5, that is $\frac{1}{3}$ of T P, makes the *pyramide* C D E P T equal to 1500, which multiplyed by 4 (because there are 4 of them) is 6000; B C, 34; C P, 12; B P, 408; this last being multiplyed by 7$\frac{1}{2}$, that is $\frac{1}{2}$ the altitude T P, makes 3060 equal to the *prisme* BCQPTS, this being doubled (because there are two of that magnitude) is 6120; P E, 25; P K, 60; P F, 1500; which being multiplyed by 7$\frac{1}{2}$ makes 11250, equal to the prisme P E F K Z T, this being doubled (because there are two opposites equal) is 22500; P Q, 34; Q L, 60; L P, 2040; this last number being multiplyed by 15 the altitude of the *pyramide* T P, produceth 30600; the parallelepipedon Q P K L V Z T S; these 4 composed numbers being added together, makes the whole *frustum pyramide*.

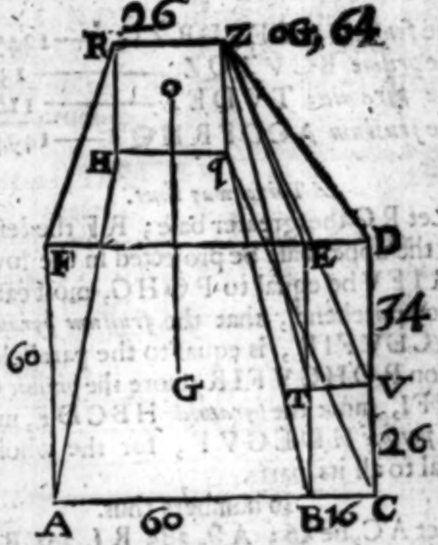
*Thus.*

The 4 *pyramides* are——————6000.
The 2 lesser *prismes* are——————6120.
The 2 greater *prismes* are——————22500.
The *parallelepipedon* is——————30600.
The whole *frustum pyramide* is——————65220.

*A Second way thus.*

Let F C be the greater base, R Q the lesser, G O the height; make it as R H : H Q :: F A : A B; draw B E parallel to F A, then will the

the plane F B be like the plane R Q; Let the Figure be completed, then will the *frustum* ACDFRHQZ be equal to the *frustum* ABEFRHQZ more the prisme, BCVT QZ, more the *pyramide* TVDEZ: here note that TB is made equal to QZ.




*The Example in Numbers.*

The *frustum* ABEFRHQZ, is equal to 124501 $\frac{1}{3}$, found by the first *Proposition*, BC, 16; CV, 26; BV, 416 $\frac{1}{2}$ being multiplyed by $\frac{1}{2}$ the altitude, that is 32, makes 13312, equal

equal to the prisme BCVTQZ; TV, 26; VD, 34; TD, 544; being multiplyed by ⅓ of GQ that is 21⅓, makes 11605⅓ equal to the *pyramide* TVDEZ; these three composed numbers being added together, makes 149418⅓ equal to the *frustum pyramide* ACD FRHQZ.

The *frustum* ABEFHRZQ——124501⅓.
The *prisme* BCVTQZ ————13312.
The *pyramide* TVDEZ————11605⅓.
The *frustum* ACDFRHQZ——149418⅓.

*A Third way thus.*

Let PC the greater base; RF the lesser; Let the upper base be projected in the lower, as RIFV be equal to POHG, most easie it is to apprehend, that the *frustum pyramide* PACEVFIR, is equal to the parallelepipedon POHGVFIR, more the *prisme* OA BHFI, more the *pyramide* HBCDF, more the *prisme* HDEGVF, for the whole is equal to all its parts.

In numbers thus.

Let AC, be 56; AP, 38; RI, 26; RV, 30; RP, 40; RI, 26; VR, 30; IV, 780 equal to PH; which being drawn into RP, 40; makes POHGVFIR, 31200: OA, 12; AB, 30; AH, 360; being multiplyed by 20, that is half the height PR, it makes

7200

7200 equal to OABHFI. BC, 28; HB, 12; BD, 336; being multiplyed by $\frac{1}{3}$ of the altitude, that is 13 $\frac{1}{3}$, it produceth 4480; equal to the *pyramide* HBCDF; GH, 26; HD, 28; HE, 728; being multiplyed by 20, the product is 14560 equal to the prisme HDEGVF; these 4 composed numbers being added together, makes the solidity of the said *frustum*.


| | | |
|---|---|---|
| The *parallelepipedon* | POHGFVRI | =31200 |
| The *prisme* | OABHFI | =07200 |
| The *pyramide* | HBCDF | =04480 |
| The *prisme* | HDEGVF | =14560 |
| The *frustum pyramide* | PACEVFIR | =57440 |

Then,

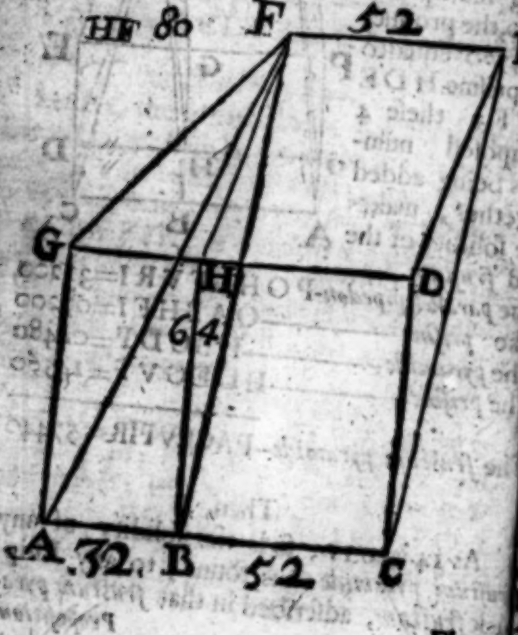
As 14 is to 11, so is the solidity of any *frustum pyramide* thus found, to any Elliptick *frustum*, adscribed in that *frustum pyramide*.

*Proposition.*

## PROPOSITION. III.

*To find the solidity of a* Frustum Prisme.

Et CAGDEF be a *prisme*, who solidity is required, let BC be ma equal to FE, and BH parallel to GD, and th




Figure

figure being completed, the *frustum prisme* ACDGFE may be composed of the *prisme* CDHFE, more the *pyramide* BAGHF: further demonstration is needless.

*An Example in Numbers.*

BC, 52; BH, 64; BD, 3328; this last number being multiplyed by 40, that is half the height, makes 133120 equal to the *prisme* BCDEF; AB, 32; BH, 64; BG, 2048: this last being drawn into $26\frac{2}{3}$, that is, a third part of the altitude makes $54613\frac{1}{3}$ equal to the *pyramide* GABHF: this *prisme* and *pyramide* being added together, makes 187733 $\frac{1}{3}$, equal to the *frustum prisme* AGD CEF.

*The second Case thus.*

Let ZBCDEH be a *frustum prisme*; Let AC be equal to HE and AO parallel to ZB, the Figure being completed, the *prisme* AC DOHE less the *pyramide* ABZOH is equal to the *prisme* BCDZHE; or thus, the *prisme* ZBODFE more the solidity ZBHF, that is, half the *pyramide* ZBAOH, is equal to the *frustum* proposed.

*Example in Numbers.*

AC, 84; AO, 64; AD, 5376; this last drawn into 40, half the height makes 215040; equal to the *prisme* OACDHE; AC, 32; BZ, 64; AZ, 2048 being multiplyed by $26\frac{2}{3}$, that is, one third of the height, it makes


546131 equal to the *pyramide* A B Z O H this *pyramide* taken from the *prisme* leaves 160426, equal to the *frustum prisme* proposed.

Then as 14 is to 11, so are such *prismes* to Elliptick solids adscribed in those *prismes*.

*Hence follows a Fourth way for finding the solidity of these irregular* frustums.

Suppose A C D F G N I H be the *frustum* proposed, because F A is equal to D C, and A C equal to D F, and G N equal to I H, and

NI to GH; and because GN is less than FA, and HI than DC; therefore if AN, CI, DH, FG, be continued towards Z and V, they will meet, suppose at Z and V, then ACDFVZ, less NIHGVZ, there remains ACDFGNIH the *frustum* proposed.


It also follows; That such *prismes* have such proportions one to another, as Squares and Cubes of their corresponding sides, disjoyned, thus.

The *pyramide* BCDEZ, is to the *pyramide* RQIHZ, as the cube of BE to the cube of RQ; 8, 12. The *prisme* FABEVZ, is to the *prisme* GNQRVZ, as the square of FA, is to the square of GN; the reason why these *prismes* are not in a triplecate *ratio* of their homologal sides, as well as the *pyramides*; is because FE, GR, and VZ are equal,

equal, and the *ratio* riseth but from V G, G N and V F, and F A; whereas in the *pyramide* the *ratio* riseth from three, that is from Z R, R Q, and R H; and from Z E, E D, and E B.

*In Numbers thus.*

Let V Z be 9, Z R, 3; Z E, 6; R Q, E B, 8; R H, 3 ½; E D, 7; Therefore G Q R Z V is equal to 54, and R Q I H Z equal to 14; then as the cube of 3, that is 27, to the cube of 6, that is 216; so is the *pyramide* R Q I H Z, that is 14, to the *pyramide* E B C D Z, that is 112. Then 112 less 14 is equal to 98, equal to R Q I H D C B E. Again, as the square of 3, that is 9, to the square of 6, that is, 36; so is the *prisme* G Q R Z V; that is, 54 to the *prisme* F A B E Z V; that is, 216. Then 216 less, 54 is equal to 162, equal to G N Q R E B A F. This *frustum*, more the *frustum* R Q I H D B C E is equal to the *frustum* G N I H D C A F equal to 260.

*Or thus.*

As 27 : 189 :: (that is the cube of Z E less the cube of Z R) 14 : 98, equal to the *frustum* R Q I H D C B E. Again, as 9 : 27 :: (that is the square of Z E less the square of Z R) 54 : 162, equal to the *frustum* G N Q R E B A F.

Wh

What ever is said of these *prismes* and *pyramides*, the same is to be understood in Cones and Ecliptick *prismes*;

For they are in duplicate and triplicate *ratio* of their homologal sides, further for as much as that, the two first terms in each proportion may be fixt; there may by the help of a Table of Squares and Cubes, the solidity of such solids easily be calculated gradually, that is, inch by inch, or foot by foot, the two fixed numbers in the *pyramide* are the Cube of the continuation, or the side R Z; and the *pyramide* R Q I H, the like in the *prisme*, and with the square of the continuation R Z.

*Thus.*

27 : 14 :: 37 : 19 $\frac{1}{27}$, 27 : 14 :: 98 : 50 $\frac{22}{27}$, 27 : 14 :: 189 : 98, these three numbers, namely, 19 $\frac{5}{27}$, 50 $\frac{22}{27}$, and 98, are the solidity of the *frustum pyramide* R Q I H D C B, the first number is the solidity of one inch, the second number of two inches, the third of three inches, for the *prisme*; as 9 : 54 :: 7 : 42, 9 : 54 :: 16 : 96, 9 : 54 :: 27 : 162; these 3 numbers, *viz.* 42, 96, and 162 are the solidity of the *frustum prisme* G N Q R E B A F, taken inch by inch; therefore 42 more 19 $\frac{5}{27}$ equal 61 $\frac{5}{27}$, 96 more 50 $\frac{22}{27}$ equal 146 $\frac{22}{27}$, 162 more 98 equal to 260, are equal to the solidity of the whole *frustum* GNIHDCAF, taken inch by inch; the like for any Elliptick *frustum*.

 *Proposition.*

### Proposition. IV.

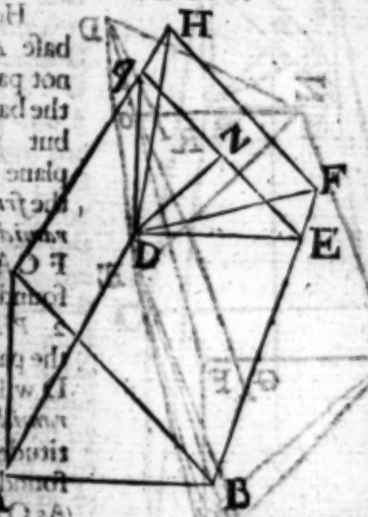
**To find the solidity of a frustum pyramide, whose Bases are not parallel; and the inclination is from side to side.**


LEt R HE TCBA[illegible] be a *fru-stum pyra-mide*, th[illegible] base RH[illegible] ET, no[illegible] parallel t[illegible] the base [illegible] ABC, [illegible] RHD[illegible] be parall[illegible] to ABC[illegible] F, the[illegible] may th[illegible] whole *fru-stum pyra-mide* R[illegible] ETABC F be com-posed o[illegible] the *fru-stum*

*stum pyramide* R H D Z A B C F more; the *prisme* R H D Z T E, by a composition of the last *Propos.* P D being the altitude of the *frustum pyramidis* R H D Z A B C F: E X the height of the *prisme* R H D Z T E may be found, thus, B D : D P :: D E : E X; a further demonstration is needless.

H.

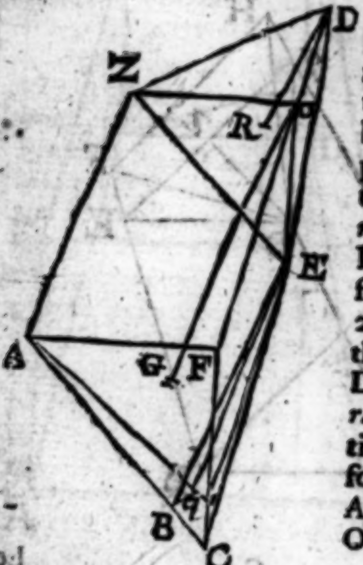
*To find the solidity of a triangular* frustum pyramide, *whose bases are not parallel.*



Let CABDHF be a *frustum pyramide*, CAB not parallel to HDF, let QDE be parallel to CAB; the triangular *frustum* QDEBAC is half a quadrangular *frustum pyramide*, found by the 2 *Proposion*, DZ the height of the *pyramide* QHFED; Therefore the *frustum pyramide* QDEBAC more the *pyramide* QHFED will be equal to the *frustum pyramide* FHDABC.

III.

*To find the solidity of the* frustum pyramide, ACFEDZ.



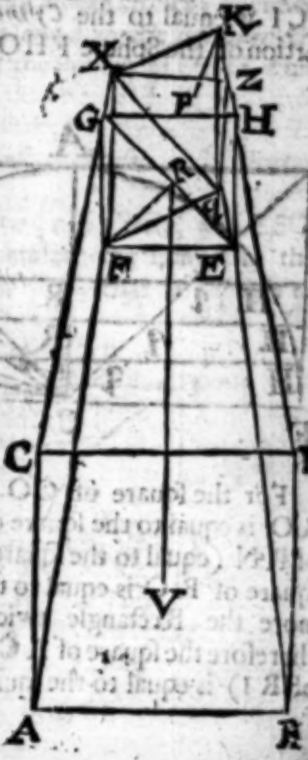

Here the base AFC is not parallel to the base ZDE but to the plane ZOE; the *frustum pyramide* ZOEFCA may be found by the 2 *Proposition*, the part ZOED will be a *pyramide*, its altitude may be found, thus, As OF: OG :: OD : DR, then

then the *frustum pyramide* ZOEFCA more the *pyramide* ZOED will be equal to the solid ACFEOZD.

IV.

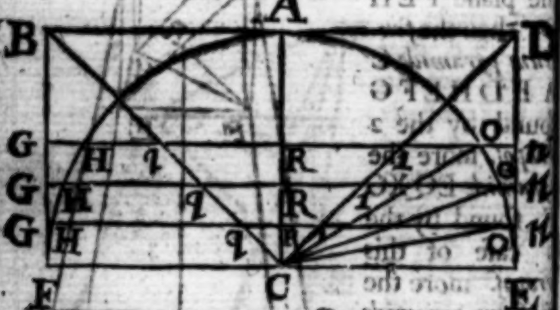
*To find the solidity of a* frustum pyramide, *whose bases are not parallel, and whose inclination is from Angle to Angle.*

Let CABDFXKQ be a *frustum pyramide*, the base CABD not parallel to the base FXKQ, but parallel to the plane FEHG, then the *frustum pyramide* CABDHEFG found by the 2 *Propos.* more the *pyramide* EQXGF, found by the 2 case of this *Propos.* more the *frustum pyramide* GEHZXQ, found by the 2 Propos. more the *pyramide* XKZQ, found by the 3 case of this *Propos.* will be equal to the whole *frustum pyramide* CABDFXKQ.



## PROPOSITION V.

LEt FHAOE be a Semicircle, HO parallel to FE; FB and ED parallel to CA, the Figure being completed, the *Cone* QCI is equal to the *Cylinder* GE less the portion of the Sphere FHOE.

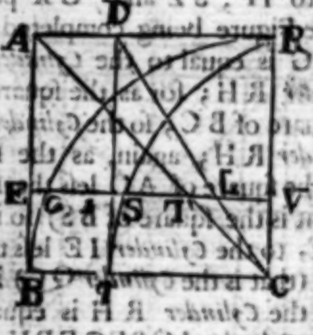



For the square of CO less the square of RO is equal to the square of RC, the square of RN (equal to the square of CO) less the square of RO is equal to the square of ON more the Rectangle twice under RON; therefore the square of RC (that is the square of RI) is equal to the square of ON more, the

the double Rectangle R O N, by 4 : 2. every one of these being considered one by one; but being collected (*viz.*) all the squares of R I is equal to all the squares of R N, less all the squares of R O; or if there be taken the quadruple of them, all the squares of Q I is equal to all the squares of G N less all the squares of H O; by 2 : 12, all the circles; therefore the *Cone* Q C I is equal to the *Cylinder* G E less the portion of the Sphere H F E O which was proposed.

*The second Case.*

Let R O B C be ¼ of a Circle, and RSQC ¼ of an Ellipsis complete the Diagram, then will the square of Q C, that is I V less the




C 4 squares

square of LV be equal to the square of S V. For A R : R D :: T V : V L, therefore the squares of them, and A R : R D :: O V : V S, therefore the squares of them; therefore, as the square of T V, to the square of V L, so the square of O V, to the square of V S; Therefore, as the square of E V, to the square of E V less the square of T V. So the square of I V, to the square of I V less the square of L V; but the square of E V less the square of T V is equal to the square of O V; therefore the square of I V less the square of L V equal to the square of S V.

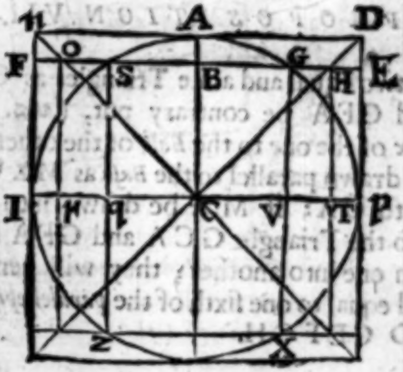
### PROPOSITION VI.

IN the Hemisphere I A P, Let F E be parallel to I P, S Z and G X parallel to A C; the Figure being completed; the *Cylinder* Q G is equal to the *Cylinder* I E less the *Cylinder* R H; for as the square of A C, to the square of B C; so the *Cylinder* I E, to the *Cylinder* R H; again, as the square of A C, to the square of A C less the square of B C, (that is the square of B S) so is the *Cylinder* I E, to the *Cylinder* I E less the *Cylinder* R H, (that is the *Cylinder* Q G) it followeth that the *Cylinder* R H is equal to the *excavatus Cylinder* I Q S F G E P V, for the *cylin-*

*Cylinder* QG is equal to the *excavetus Cylinder* IROFHTPE.

But if the *excavetus Cylinder* ORQSGVTH, be added to both; the *Cylinder* RH will be equal to the *excavetus* or hollow *cylinder* QIFSGVPE; but the *cone* OCH is equal to FISGPE by the first part of the Fifth Proposition; Therefore IQSGVP is equal to ORCTH, that is, $\frac{2}{3}$ of the difference betwixt the circumscribed IE and inscribed QG *cylinders*; substracted from the circumscribed *cylinder* IE is equal to the portion of the Sphere ISGP; or if $\frac{1}{3}$ of the difference be added to the inscribed *cylinder* QG it will be equal to the portion of the Sphere ISGP.

The like in a Spheroid by the second part of the last Proposition.



*In Numbers thus.*

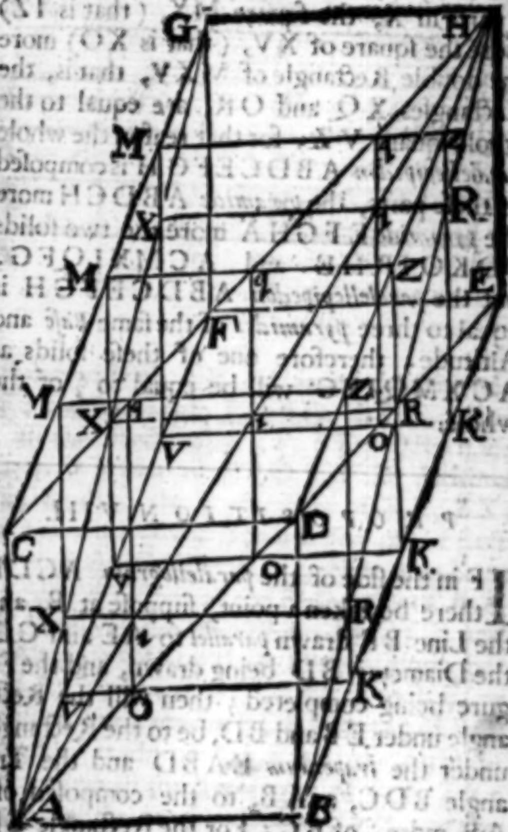
Let IP be 20; QV, 16; CB, 10; the square of IP is 400, the square of QV 256; the difference of the squares is 144, a third part of that number is 48, which substracted from the greater square 400, leaves 352: or if two thirds of the difference, that is, 96 be added to the lesser square, that is, to 256, the sum will be 352 the true mean square; but if it be as 14 : 11 :: 352 : 276 $\frac{4}{7}$ the mean circle, which being multiplyed by the length ZS, 20; the Product is 5531$\frac{3}{7}$ the solidity of the portion ZISGPX.

---

## PROPOSITION VII.

IF two equal and alike Triangles as GCA and GFA be contrary put, (*viz.*) the *Vertex* of the one to the *Base* of the other, and Lines drawn parallel to the *Base* as MXV parallel to CA; If MX be drawn into XV, that so the Triangle GCA and GFA being drawn one into another; they will generate a solid equal to one sixth of the *paralellepipedon* ABDCEFGH.

For




For by 45. 2. the Line MV being cut into [illegible] (that is 12) more the square of XV, (that is XO) more the [illegible] Rectangle of MX, XV, that is, the Rectangles XQ and OR are equal to the whole [illegible] the whole parallelepipedon ABDCEFGH is composed of [illegible] ABDCH more [illegible] EFGHA more two solids [illegible] IOFG. [illegible] parallelepipedon ABDCEFGH is equal to [illegible] the same base and Altitude, therefore one of these solids as ACO[illegible] will be equal to [illegible] of the wh[illegible]

PROP. VIII.

If in the side of the parallelogram [illegible] there be taken a point [illegible] and the Line EF be drawn parallel to [illegible] CD, the Diameter BD being drawn, and the figure being compleated; then will the Rectangle under E and BD, be to the Rectangle under the [illegible] BD and [illegible] angle BDC, [illegible] to the compo[illegible] AB more [illegible] of BC; For the rectangle [illegible] BD; that is, all the Rectangles of AB, BC; [illegible] EX, EF, ED. That is the parallelepipedon

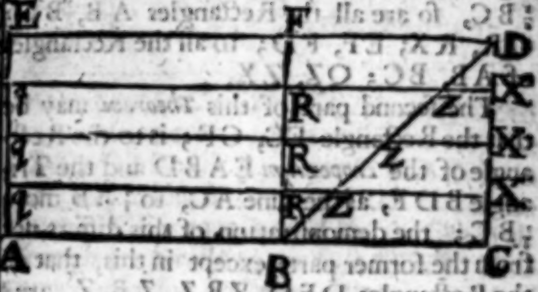
For

For by 4 ; 2. the Line MV being cut into 2 parts in X, the square MX, (that is IZ) more the square of XV, (that is XO) more the double Rectangle of MXV, that is, the Rectangles XQ and OR, are equal to the whole square VZ; for that reason the whole *paralellepipedon* ABDCEFGH is composed of these parts, the *pyramide* ABDCH more the *pyramide* EFGHA more the two solids BDKOIRHE and ACMXIQFG, but the *paralellepipedon* ABDCEFGH is equal to three *pyramides* of the same *Base* and Altitude; therefore one of these solids as ACXMQIFG will be equal to $\frac{1}{3}$ of the whole.

---

PROPOSITION VIII.

If in the side of the *paralellogram* ACDE there be taken a point, suppose at B, and the Line BF drawn *parallel* to AE and CD, the Diameter BD being drawn, and the Figure being completed; then will the Rectangle under EB and BD, be to the Rectangle under the *trapezium* EABD and the Triangle BDC, as AB, to the composed of; AB more $\frac{1}{2}$ of BC; For the Rectangle EB, BD; that is, all the Rectangles of AB, BC; QB, RX; EF, FD; that is the *paralelle-pipedon*




pipedon made of A B, B C; and the Altitude B F: the Rectangle E A B D, B D C; that is, all the Rectangles E F, F D, Q R, R Z, that is a *prisme* of E F D and the Altitude F B; more all the Rectangles of R Z, Z X; but a *paralellepipedon* is to a *prisme* of the same *Base* and Altitude as 1 to $\frac{1}{2}$, that may be as A B, to $\frac{1}{2}$ A B, but all the Rectangles of R Z, Z X, are $\frac{1}{3}$ of the *paralellepipedon* B C B and the Altitude C D by the last Proposition; but the *paralellepipedon* B C B F D, is to B C, as $\frac{1}{2}$ of the *paralellepipedon* B C B F B, to $\frac{1}{2}$ of B C, the *prisme* made of D F E and the Altitude E A, more all the Rectangles of R Z, Z X; that is $\frac{1}{3}$ of the *paralellepipedon* B C B, and the Altitude B F, is equal to the Rectangle E A B D, B D C; that is all the Rectangles of

Q Z X;

QZX; Therefore as AB, to $\frac{1}{2}$ AB mo[re] $\frac{1}{3}$ BC, so are all the Rectangles AB, BC; QR, RX; EF, FD; to all the Rectangl[es] of AB, BC; QZ, ZX.

The second part of this *Theorem* may b[e] that the Rectangle EC, CF; is to the Rec[t]angle of the *Tropezium* EABD and the Tr[i]angle BDF, as the Line AC, to $\frac{1}{2}$ AB mo[re] $\frac{1}{3}$ BC; the demonstration of this differs n[ot] from the former part, except in this, that the Rectangles DFD, ZRZ, ZRZ, are of the *paralellepipedon*, as appears by the la[st] Proposition.

This Proposition is the 30 of the second [of] *Cavul. Geomet. Indivis.* and is of good use [in] *solid Geometry.*

## PROPOSITION. IX.

LEt ADBEK be a *cone* and be cu[t] through the *axis*, whose Section will b[e] the Triangle AKB, Let it be cut by anothe[r] Plane as EZD; the diameter of the Sectio[n] ZC parallel to BK, the *Base* of the Sectio[n] ED at Right Angle at C with AB; Let it b[e] as ABA : AKB :: PZ : ZK, then will PZC be equal to the square of CE;

For

For,

AB : BK :: AC : CZ, } therefore by 23. 6.
BA : AK :: CB : ZK, }

ABA:BKA::ACB:CZK::PZ:ZK, apply CZ to the two last terms, aud it will be as ACB: CZK :: CZP : CZK, because CZK is in each proportion; Therefore ACB that is the square of CE is equal to CZP; the like in all other planes *parallel* to the *Base* AEBD. This manner of section *Apollonius* calls a *parabola*, but


if

if the *semiparabola* E I Z R C be moved about the *axis* ZC, until it return to that point from whence it began its motion, it will *Generate a solide*, *Archimedes* names such a solid, a Rectangled *conoide*; but generally it is called by the name of a *parabolical conoide*.

## PROPOSITION X.

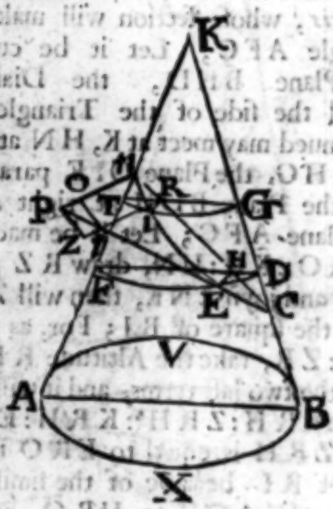
LEt A V B X K be a *cone*, and be cut through the *axis*, whose Section shall be the Triangle AKB, Let it be cut by another Plane as N I E C whose diameter is N C, (being continued if need be) shall cut both sides of the *cone*, the Plane N I E C being at Right Angle with the Plane AKB, the Planes F E D and T I G parallel to the Plane A V B X, Let it be made, as C R N : G R T :: C N : N P, then will Q R N be equal to the square of R I; for C N : N P :: C R : R Q, taking the Altitude R N it may be C N : N R :: C R N : Q R N :: C R N : G R T, in these last four terms C R N is twice, therefore Q R N is equal to G R T, that is the square of R I.

By reason of the similitude of the Triangle C H D to C R G, and of the Triangle N R T to N H F, it may be, as

NR : TR :: NH : FH } by 23. 6.
RC : RG :: HC : HD }

NRC : TRG :: NHC : FHD, but TRG is equal to the ſquare of RI, and FHD equal to the ſquare of HE; therefore as NRC to the ſquare of RI, ſo is NHC to the ſquare of HE.

This Section of a *cone* is named by *Apollonius* an *Ellipſis*.



But if the *Semi-Ellipsis* CEIN be move[d] about the *axis*, CN, until it return to th[e] point from whence it began its motion, it wi[ll] Generate a solid, *Archimides* calls such a soli[d] a *Spheroide* ; but if it be turned about th[e] lesser Diameter, then he calls that solid a *Sphe*-*roide prolatus*.

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*PROPOSITION* XI.

LEt ABCBF be a *cone*, and be cut throug[h] the *axis*, whose section will make th[e] the Triangle AFC, Let it be cut b[y] another Plane BHD, the Diamete[r] HG, and the side of the Triangle C[F] being continued may meet at K, HN at righ[t] Angle with HG, the Plane OIE parallel t[o] ABCD, the Plane BDH at right Angl[es] with the Plane AFC; Let it be made, a[s] KRH:ERO::KH:HN, draw RZ paral-lel to HN, and joyn ZNK, then will ZRH be equal to the square of RI; For, as KH: NH::KR:ZR, take the Altitude RH and apply it to the two last terms, and it will be a[s] KH:NH::KRH:ZRH::KRH:ERO, Therefore ZRH is equal to ERO that i[s] the square of RI because of the similitude of the Triangle AGH to HRO, and o[f] KGC to KRE, it may be as,

KG:

$$\left.\begin{array}{l} KO : CG :: KR : ER \\ GH : GA :: RH : RO \end{array}\right\} \text{by } 23, 6.$$

---

KGH : CGA :: KRH : ERO; CGA is equal to the square of GB, and ERO is equal to


 the

the square of R I; Therefore as K G H, the square of G B, so is K R H, to the squa
of R I.

This manner of Section *Apollonius* calls
*Hyperbola*; but if the *semihyperbola* B I H R
be moved about the Axis H G, until it retu
to that point from whence it began its m
tion, it will Generate a solid; such a solid
called by *Archimedes* an *Amblygon conoid*
but generally it is called a *Hyperbolical co*
*ide*.

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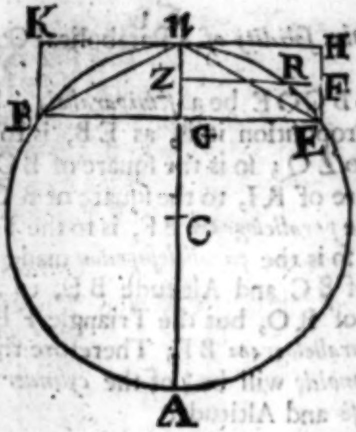
## PROPOSITION XII.

*To find the solidity of the portion of the Sphere* B N R E.

LEt A B N E be a *Sphere* and B H a C
*linder*, it is manifest that the *paralell*
*p*...*don* whose *Base* is the square of G E, a
Altitude G N is equal to the *paralellepipe*
whose *Base* is the square of G N and Altitu
A G; it is also manifest that the Rectan
A G N is equal to the square G E, the l
for any other, as, A Z N equal to Z R
Therefore by the 8 Proposition, as A G t
A G more ⅓ G N, so are all the squares Z
to all the squares of Z R; Therefore, as A
to ⅓ A G more ⅓ G N, so the *Cylinder* E

to the portion of the Sphere ERNB. The like in a *Spheroide*.




*In Numbers thus.*

Let AG be 150; GN, 54; GE 90; the *paralellepipedon* made of the ſquare of BE and Altitude GN, is 1749600; then, as AG, 150; to ½ AG, more ⅙ GN; that is 75 more 9 that is 84; ſo is 1749600, to 979776, that is all the ſquares in the portion of the Sphere BNRE; then as 14 : 11 :: 979776 : 769824, the portion of the Sphere BNRE.

 Propoſ.

## PROPOSITION XIII.

*To find the solidity of a* Parabolical Conoid[e]

Let BCOE be a *semiparabola*; by th[e] 9 Proposition it is, as EB, is to ER that is to ZQ; so is the square of BC that [is] the square of RI, to the square of RO, th[at] is, as the *parallelogram* BF, is to the Triang[le] FEB; so is the *parallelepipedon* made of th[e] square of BC and Altitude BE, to all th[e] squares of RO; but the Triangle FBE is [½] of the *parallelogram* BF; Therefore the *para-bolical conoide* will be ½ of the *cylinder* of th[e] same *Base* and Altitude.


*Thus*

*Thus in Numbers.*

Let AB, 10; BC, 10; Therefore AC, 20; Let BE, 30; the square of AC, 20, is 400, which being multiplyed by ½ EB, that is by 15; the Product is 6000, equal to all the squares in the *conoide* AEC; but if it be made as 14 : 11 :: 6000 : 4714 ², the solidity of the *conoide* AEC.

---

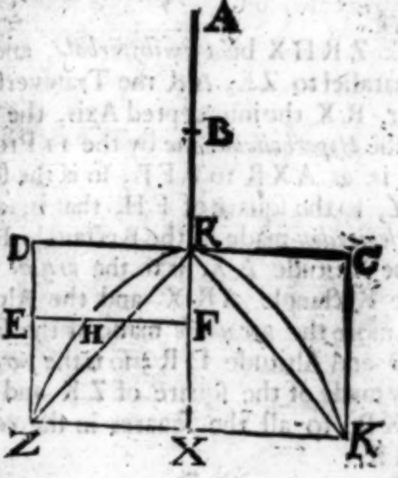
PROPOSITION. XIV.

*To find the solidity of a* Hyperbolical Conoide.

Let ZRHX be a *semihyperbola*, and EF parallel to ZK, AR the Transverse Diameter, RX the intercepted Axis, the point H in the *Hyperbolical* Line by the 11 Proposition it is, as AXR to AFR, so is the square of XZ, to the square of FH, that is, as the *parallelepipedon* made of the Rectangle RXZ and the Altitude AX, is to the *prisme* made of the Rectangle ARX and the Altitude AD, more the *pyramide* made of the square of ZD and Altitude DR; so is the *parallelepipedon* made of the square of ZK and Altitude XR, to all the squares in the *conoide* ZHRK.

 But

But as the *parallelepipedon* made of the Rect-angle RXZ and Altitude AX, is to the *prisme* made of the Rectangle ARX and Al-titude RD, more the *pyramide* made of the square DZ and Altitude DR; so is the Line AX to the Line BR (that is $\frac{1}{2}$ AR) more $\frac{1}{3}$ of the Line RX, by the 8 Proposition; Therefore by the 11. 5. it will be as the Line AX, is to the Line BR, more $\frac{1}{3}$ of the Line RX, so is the *parallelepipedon* made of the square of ZK and Altitude XR, to all the squares in the *conoide* ZHRK; then as 14:11, so are all the squares in the *conoide* ZHRK to the *conoide* ZHRK.




Let

*In Numbers thus.*

Let AX be 150; RX, 54; ZX, 90; AR, 96; BR, 48; the *parallelepipedon* whose *Base* is the Rectangle of RXZ and the Altitude AX is 729000.

The *prisme* whose *Base* is the Rectangle ARX and Altitude RD is 233280.

The *pyramide* whose *Base* is the square of RX and Altitude RD is 87480.

The *parallelepipedon* whose *Base* is the square of ZX and Altitude XR is 437400.

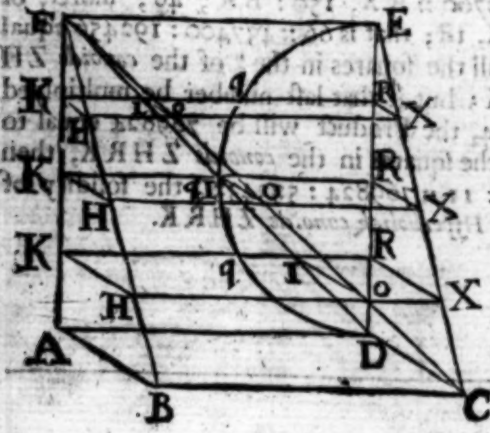
Then 729000 : 233280 more 87480, that is, 320760 :: AX, 150 : BR, 48, more $\frac{1}{3}$ of RX, 18; that is 66 :: 437400 : 192456 equal to all the squares in the $\frac{1}{4}$ of the *conoide* ZHRK; but if that last number be multiplyed by 4, the Product will be 769824 equal to all the squares in the *conoide* ZHRK, then 14 : 11 :: 769824 : 533433 $\frac{1}{7}$ the solidity of the *Hyperbolick conoide* ZHRK.

PROP.

## PROPOSITION XV.

*To find the solidity of a whole Sphere, or a portion thereof.*

LEt ADEF be a Plane and the Semicircle DQIQE be in that Plane, Let the Planes ABCD and CDE be at Right Angle with the Plane ADEF; Let DC, AB and EF each of them be equal to DE;



Let the Planes K H X R be parallel to the *Base* of the *prisme*; namely, parallel to A B C D. The Rectangle D R E is equal to the square of R Q, by 35 3, that is, the Rectangle comprehended of I R and R X, for R X is equal to R E, and I R is equal to R D, therefore all the squares in ½ of the Sphere E Q I Q D are equal to all the Rectangle in the *prisme* C D F E, the like in any part: Therefore the *prisme* R X H K P E, less the *pyramide* O I K H F shall be equal to all the squares in the portion of the Sphere Q E R. Then as 14 : 11, so the *prisme* X R I O F E to the solidity of the portion of the Sphere Q E R,

*In Numbers thus.*

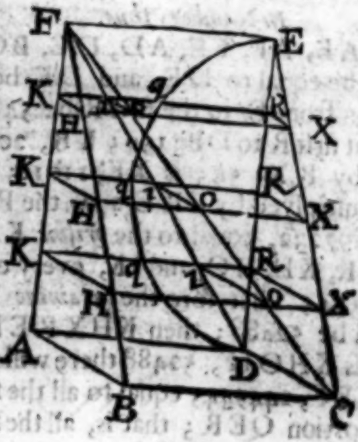
Let A B, A F, F E, A D, D C, B C each of these be equal to D E, and D E be equal to 204: from E to the first R be 54, and from that first R to D be 150; K R, 204 multiplyed by R X, 54; is X K, 11016; which being multiplyed by ½ R E 27; the Product will be 297232, equal to the *prisme* K H X R E F; F K, K H, H O, and I K, every of them equal to 54; Therefore the *pyramide* K H O I F will be 52488; then K H X R E F, 297232; less K H O I F, 52488 there will remain I O X R E F, 244944; equal to all the squares in the portion Q E R; that is, all the squares in the portion of ½ of a Sphere,

PROP.

## PROPOSITION XVI.

*To find the solidity of a whole* Spheroide, *or a portion thereof.*

LEt FADE be a Plane, the *Ellipsis* EQD be in that Plane; Let AB and DC each of them be equal to DE, the Planes KRXH parallel to the Base of the *prisme* ABCD. Find the Line FE by the 10th. Prop. by the said 10th. Prop. the Rectangle IRE




will

will be equal to the square of R Q; but the Rectangle I R E is equal to the Rectangle I R X, because D C is equal to D E, therefore R X is equal to R E, therefore the Rectangle I R X is equal to the square of R Q; therefore the *prisme* X H K R E F less the *pyramide* K H O I F will be equal to all the squares in the portion Q E R.

*In Numbers Thus.*

Let D E the longest Diameter be 36, the Conjugate or shortest 18, therefore F E will be 12, for it is 36 : 18 :: 18 : 12. Let from E to the first R be 9, then F E, 12, multiplyed by R X, 9; X K will be 108; which being multiplyed by $\frac{1}{2}$ of E R, that is 4 $\frac{1}{2}$ the Product will be 486, equal to K H X R E F; for the finding of K I, work thus, A F, 36; to A D, 12; so is F K, 9; to I K, 3; then I K, 3, multiplyed by K H 9; the Product is 27, that is, H I; this last number being multiplyed by $\frac{1}{3}$ of K F, that is by 3; the *pyramide* K H I O F will be equal to 81, which being substracted from the *prisme* K H X R E F there will remain 405 the *prisme* I O X R E F equal to all the squares in the portion of the *spheroide* Q E R.

PROP.

PROPOSITION XVI.

*To find the solidity of a hyperbolical Conoide.*

LEt MAEL be a Plane; and the *hyperbola* be in that Plane; to that Plane let the Planes BAM, CEL and DEL be perpendicular; Let DE, CF and BA be each of them equal to AM; Let the Planes KIXH be parallel to the Plane ABDE the *Base* of the *prisme* ABDELM; find the Lines MD and MG by the 11 Proposition, the Rectangle IKM, is equal to the square of KZ, but the Rectangle IKM is equal to the Rectangle IKHX, because AB is equal to AM, therefore HK is equal to KM; Therefore the Rectangle IKHX, is equal to the square KZ; Therefore the *prisme* ABDELM, is equal to all the squares in the *conoide* AMZ.

*In Numbers thus.*

Let GM the Transverse Diameter be 18; MA, 12; AE, 10; ML, 6; AB equal to AM multiplyed by AE, 16; the Product is 72 equal to ABCF, which being multiplyed by ½ AM, that is, 6, the Product is 432 equal to the *prisme* ABCFLM; FE, 4; multi-


G
M
L
K
H
K
O
X
H
A
F
E
Z
B
C
D

multiplyed by F C, 12; the Product is 48, equal to F C D E, which being multiplyed by ⅓ of F L, that is, 4; the Product is 192, equal to the *pyramide* F C D E L; this *pyramide* being added to the *prisme* before found, the whole *prisme* A B D E L M, will be 634, equal to all the squares in the *conoide* A Z M.

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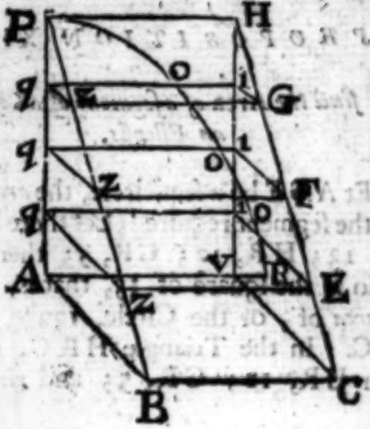
## PROPOSITION XVIII.

*To find the solidity of a* Conoide parabola.

LEt A V H P be a Plane, the *parabola* P O R A be in that Plane, Let V C and A B each of them be equal to A P; Let the Planes Q G, Q F, and Q E be parallel to the Base of the *prisme* A B C V. By the 9 Proposition the *parallelogram* H Q is equal to the square of Q O, but the Plane H Q is equal to the Rectangle Q Z G I, because I Q is equal to H P and Q Z is equal to Q P; Therefore the Plane Q G is equal to the square of Q O, the like in the rest; Therefore the *prisme* A B C V H P is equal to all the squares in the *conoide* R P A.

In




*In Numbers thus.*

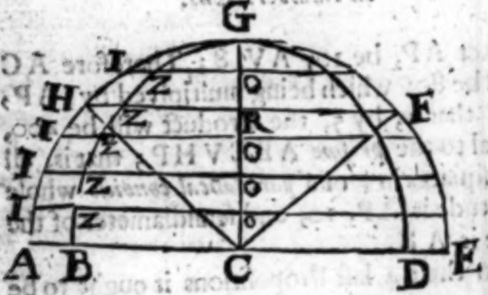
Let AP, be 10; AV, 8; Therefore AC will be 80; which being multiplyed by ½ AP, 10; that is, by 5, the Product will be 400, equal to the *prisme* ABCVHP; that is, all the squares in ½ of a *parabolical conoide* whose Altitude is AP, 10; and semidiameter of the Base is AR.

In these 4 last Propositions it ought to be made, as 14 : 11; so are all the squares in the portion to the portion it self.

## PROPOSITION XIX.

*To find the* Area *of a segment of a* Circle *or an Ellepsis.*

LEt AGE be a semicircle, the *area* HGF the segment required; Let there be given AC, 13; HR, 12; CR, 5; then, as 14 to 11, so is the square of 13, that is, 169, to the *area* of $\frac{1}{2}$ of the Circle, $132\frac{11}{14}$, the *area* AGC. In the Triangle HRC, there is given HR, 12; CR, 5; and the Righ




Angl

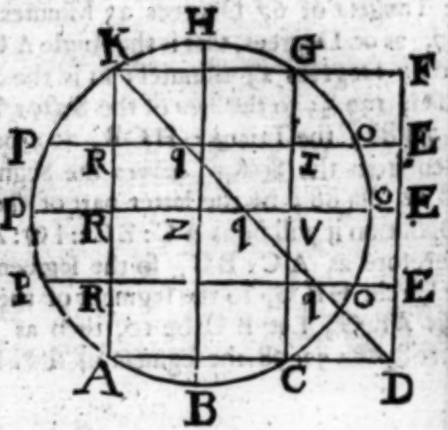
Angle CRH; therefore there is given the Angle CRH; thus, 5 : 12 :: 100000 : 240000, the Tangent of 67 Degrees 23 Minutes, again, as 90 Degrees, that is the Angle ACG, is to 67 Degrees 23 Minutes; so is the *area* ACG, 132 $\frac{11}{14}$; to the *area* of the Sector HCG, 98⌊88; the Triangle HCR, 30; being taken from the Sector, leaves the Segment HRG, 68⌊88; by the latter part of the 10 Proposition it will be as AC : BC :: IO : ZO, Therefore as AC : BC, so the segment of the Circle HRG, to the segment of the Ellipsis ZRG; Let BC be 10, then as 13 : 10 :: 68⌊88 : 52⌊98 the segment of the Ellipsis.

---

## PROPOSITION XX.

### *To find the solidity of a Circular or Elliptick Spindle.*

Let ABCOHP be a Circle, HB and PVO be two Diameters at right Angles Z; Let AK and GC be parallel to HB, the Lines PO parallel to PVO; Let GC equal to AK be the *axis*, VO the Diameter of the Spindle. Suppose there be a Solid whose *Base* shall be the *segment* KHGOBA and Altitude the *segment* APK; such




a Solid is equal to all the squares in $\frac{1}{4}$ of Sphere whose Diameter is A K; for the Rectangle O R P is equal to the Rectangle AR 35, 3, and by the same 35, 3, the Rectangl A R K are equal to all the squares in $\frac{1}{4}$ of Sphere whose Diameter is A K.

If from the Solid whose *Base* is K H G C B A and Altitude A P K, there be tak a Solid, whose *Base* is K G C A and Altitud A K P there will remain a Solid whose *B* and Altitude is equal to the *segment* G O or A P K; equal to all the squares in the $\frac{1}{4}$ the Spindle.

By the 15 Propoſition find all the ſquares in $\frac{1}{4}$ of a Sphere whoſe Diameter is AK.

By the 19 Propoſition find the *ſegment* A P R which multiplyed by K G makes a *parallelepipedon*, which being taken from $\frac{1}{4}$ of the forementioned Sphere, leaves all the ſquares in $\frac{1}{4}$ of the Spindle, this $\frac{1}{4}$ being multiplyed by 4 gives all the ſquares in the Spindle; Then, as 14 : 11, ſo are all the ſquares in the Spindle, to all the Circles, that is the Spindle it ſelf.

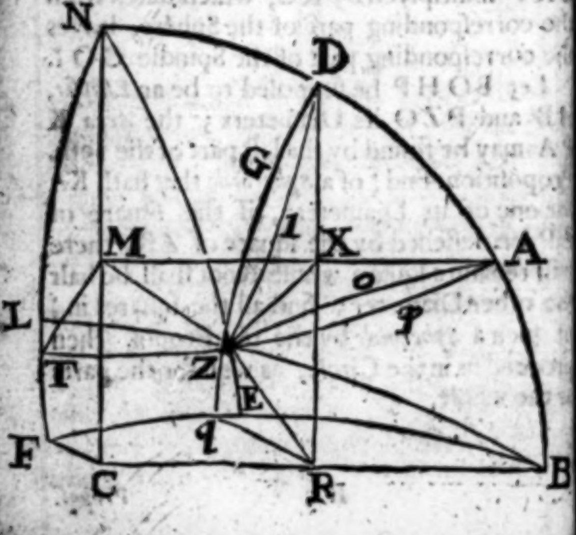
Further, as the whole ſo the parts are calculated; find the *ſegment* of a Sphere by the 15 Propoſition, and the ſegment of the Circle K R P by the 19 Propoſition, this *ſegment* K R P multiplyed by K G, which taken from the correſponding part of the Sphere, leaves the correſponding part of the Spindle G O I.

Let B O H P be ſuppoſed to be an *Ellipſis*, H B and P Z O its Diameters; the *Area* K P A may be found by the laſt part of the 19th. Propoſition: find $\frac{1}{4}$ of a *Spheroide* that hath KA for one of its Diameters, if the ſquare of Z P be leſſened by the ſquare of Z R, there will remain a ſquare whoſe Root ſhall be half the other Diameter; find all the ſquares in $\frac{1}{4}$ of ſuch a *Spheroide* by the 16. Propoſ. then proceed as in the Circle, as well for the parts as the whole.

## PROPOSITION XXI.

*To find the solidity of the second Sections in a* Sphere *or* Spheroide.

LEt D A B R C M N L F Q be ¼ of a Sphere cut by two Planes, *viz.* the Planes R X D G Z Q and M X A P Z T being at right Angles, their Intersection X Z. M A Equal to M Z; R D equal to R Z; C N equal to C B the semidiameter of the Sphere; M X and X R being known, the rest may be found thus.



1.

The *Areas* ZXD and ZXA may be found by the 19th. Proposition.

2.

The Spherical *Superficie* of BDZQ is equal to a Surface whose *Base* is equal to the Line FLN and Altitude RB.

Or,

The Spherical *Superficie* of RDZQ is equal to the *Area* of a Circle whose Diameter is equal to a Line drawn from B to D.

The like for the Spherical *Superficie* of NAZT. *Archimedes* 36. 1. of the *Sphere* and *Cylinder*.

3.

Let RDGZQ and MAPZT be Quadrants of lesser Circles of the same Sphere; RD and MA their Semidiameters; CBZE and CNZE Quadrants of great Circles of the Sphere; AOZ and DIZ Arches of great Circles of that Sphere, the Arch NZ equal to the Arch NA, and the Arch BZ equal to the Arch BD; the right lined Angle DRZ equal to the Spherical Angle DBZ, the Angle ZMA equal to the Angle ANZ.

4.

As 90 Degrees, is to the degrees and parts of a Degree in the Angle DBZ; so is the Spherical *Superficie* BADGZQ, to the Spherical *Superficie* of the Triangle BADGZ.

5.

In the Triangle I Z B D I 5 there is given the Arches B D and B Z, and the Angle D B Z; Therefore there are given the Angles B D Z and B Z D. The like in the Triangle O Z N A O for the Angles N A Z and N Z A, by the third Case of Oblique angled Spherical Triangles.

6.

In the Triangle A O Z I D, there is given the Arch A D and the Angles Z A D and Z D A; Therefore there is given the Angle A O Z I D, by the 8th. Case of Oblique angled Spherical Triangles.

7.

In the Triangle A O Z I D there are given the three Angles, Therefore the *Area* may be found, thus. As 180 Degrees, is to the excess of the three Angles over and above 180 Degrees; so is the *Area* of a great Circle of that Sphere to the *Area* of that Triangle, *Foster, Miscel.* page 21.

Or,

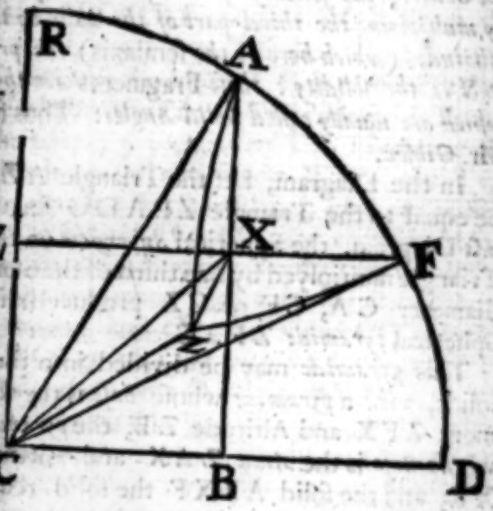
It may be found by that method which is delivered by that Learned Mathematician Mr. *John Leek*, in page, 116, of Mr. *Gibsons Syntaxis Mathematica*, Which is this. If the excess of the three Angles above 180 Degrees be multiplyed by half the Diameter of the Sphere, the *Superficies* of any Spherical Triangle is thereby produced.

By

By this Rule and by what Mr. *Gibson* delivered in that 116 page, I found this way to resolve this Proposition.

8.

By this last Rule we are to find the *Areas* of the Triangles B DIZ and NAOZ: the difference between the *Areas* of these, and those found by the 4. of this sheweth the *Areas* of the two Figures, *viz*. GZID and OZPA, which added to the *Area* DIZOA gives the *Superficie* of the mixt Lined Triangle APZGD.




9.

9.

In the 9th. Chapter of the forementioned *Syntaxis Mathematica* there is given some Rules about some solids; but when it was time to treat of these second fragments, that Author breaks off thus; *There may be other parts of a* Sphere *besides those which are here called* Fragments, (*not to speak of those which are irregular and multiform*) *which are either* Cones *or* Pyramides, *whose* Bases *lie in the superficies of the* Sphere, *and their vertices at the Center, the solidity of one of these is found by multiplying the third part of the* Base *by the Altitude (which here is the* semiaxis) *the product is the solidity: these* Fragments *are those which are usually called Solid Angles.* Thus far Mr. *Gibson.*

In the Diagram, Let the Triangle A Z F, be equal to the Triangle Z P A D G in the last Diagram, the Spherical *Superficie* of that Triangle multiplyed by one third of the Semidiameter C A, C F or C Z produceth the Spherical *pyramide* Z F A C.

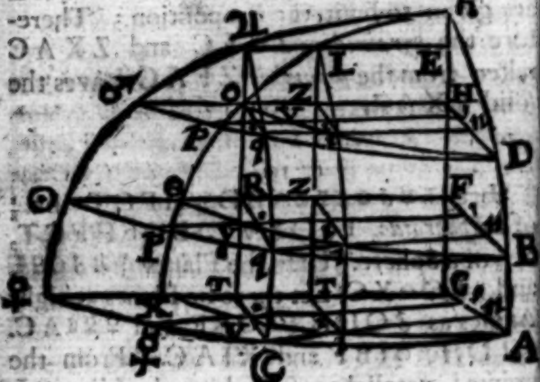
This *pyramide* may be divided into three solids, *viz.* a *pyramide* whose *Base* is the segment Z F X and Altitude Z B, the *pyramide* whose *Base* is the *Area* Z A X and Altitude X E; and the solid A Z X F the solid required. But the *pyramides* are given, because the

the *Areas* Z X F and Z X A may be found by the I of this, and the Altitudes X B and X E are given to limit the Proposition ; Therefore the *pyramides* Z X F C and Z X A C taken from the *pyramide* Z F A C leaves the solid F X A Z.

10.

Let ♄♃♂☉♀☿☾A B D ♄ H F C T be ; of a *Spheroide*, ♄☉O X V I A B D ♄ H F C T, be ; of a Sphere. Unto the Planes ♄♃♂☉♀☾ and ♄L O☉X C, Let there be Planes at right Angles as ♂Q D H, ☉Q B F and ♀☿☾A C. O I D H, ☉I B F and X I A C. From the points L parallel to ♀C draw the Line ♃L E, because the Line ♃L E is parallel to ♀C, Therefore the Line ♃T is equal to L T, and the Line T☾ is equal to T I, Therefore the Quadrants of the Circle T♃☾ and T L I are equal. In these Planes, *viz.* ♂Q D H, ☉Q B F and ♀☾A C, Let there be Lines drawn, *viz.* P comma, and Q N, parallel to ♂H. P comma, and Q N parallel to ☉F. ☿ comma, and ☾N parallel to ♀C. The points ♂P Q D, ☉P Q D and ♀☿☾A in those *Ellipses*, the points O V I D, ☉V I B and X V I A in those Circles.

♀C




♀C : XC :: ♂H : OH, by the 10. Prop.

♀C : XC :: ♃E : LE,

♂H : OH :: ♃E : LE, 11. 5.

♂H-OH : OH :: ♃E-LE : LE, 17. 5.

♂O : OH :: ♃L : LE, 19. 5.

---

☉F : ♃E :: ☉F : LE, 10th. Prop.

☉F-♃E : ♃E :: ☉F-LE : LE, 17. 5.

☉R : ♃E :: ☉Z : LE, 19. 5.

---

☉R+♂O : ♃E :: ☉Z+OZ : LE, 12. 5.

---

♃E : LE :: ♀C : XC,

♀C : XC :: ☉R+♂O : ☉Z+OZ, 11. 5.

---

♀C : XC :: *Area* R☉♀♃ : *Area* Z☉OL,

Again,

Again.

♂H : ♃E ::(QN)OH:IN, 10. Prop.
♂H-QN:QN::OH-IN(ZH)IN, 17.5.
♂O : OH::OZ : ZH, 19.5.

P, : QN::V, : IN,
P,-QN:QN::V,-IN:IN, 17.5.
P. : QN::V, : IN, 19.5.

♂O+P. : QN ::(♃E)OZ+VI:IN; (LE,) 12.5.
♂O+P.: ♃E::OZ+VI : LE,
♀C : XC::♃E : LE,
♀C : XC::♂O+P. : OZ+VI, that is,
♀C:XC::*Area* OQP♂: *Area* ZIVO. the Plane ☉PQBF or any other drawn parallel to these, shall have the same qualifications; which makes us conclude that the whole Section ILTX shall be to the whole Section ☾♃T♀ as XC is to ♀C, and as XC to ♀C so the segment ILZO to the segment Q♃O♂, the second Section in the *Spheroide*.

*PROP.*

## PROPOSITION XXII.

IF four Numbers be in proportion, that is, as the first is to the second; so is the third, to the fourth. It will always be, as the first, is to a Geometrical mean proportion between the first and second; so is the third, to a Geometrical mean proportion between the third and fourth.

Let it be,

A : B :: C : D, multiply the two first tearms by A, and it will be, as A A : A B :: C : D, 17. 7. If the two last tearms be multiplyed by C, it will be A A : A B :: C C : C D, then, as A : √A B :: C : √C D, 22. 6.

*In Numbers thus.*

4 : 9 :: 16 : 36.

16 : 36 :: 16 : 36.

16 : 36 :: 256 : 576.

4 : 6 :: 16 : 24.

PROP.

## PROPOSITION XIII.

*To find the Relation of one Hyperbola to another.*

LEt FLBC be a *semihyperbola*, its Transverse Diameter FH, its intercepted *Axis* FC. Let GMBC be another *semihyperbola*, its Transverse Diameter KG, its intercepted *Axis* GC.

Let it be made, as,

1.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| KG | : | GC | :: | HF | : | FC, | |
| KG+GC | : | GC | :: | HF+FC | : | FC, | 18. 5. |
| KC | : | GC | :: | HC | : | FC | |
| KC | : | HC | :: | GC | : | FC, | 16. 5. |
| KC | : | HC | :: | $\frac{GC}{2}$ | : | $\frac{FC}{2}$, | 17. 7. |
| KC | : | HC | :: | GF | : | FE | |
| KF | : | HE | :: | GF | : | FE, | 19. 5. |
| KC | : | HC | :: | KF | : | HE, | 11. 5. |
| GC | : | FC | :: | GF | : | FE, | 11. 5. |
| KCG | : | HCF | :: | KFG | : | HEF, | 23. 6. |
| KCG | : | KFG | :: | HCF | : | HEF, | 16. 5. |
| HCF | : | HEF | :: | CBC | : | ELE, | 11 Pro. |
| KCG | : | KFG | :: | CBC | : | FMF=ELE, | 11. 5. 11, Prop. |

Therefore

Therefore FM is equal to the Line LE, and for as much as GC is divided in F, in the same *Ratio* as FC is in E, Therefore,

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| GG | : | FC | :: | GF | : | FE, | 17. 7. |
| GC | : | FG | :: | FC | : | EC, | 19. 5. |
| GC | : | FC | :: | MD | : | LD, | |
| GC | : | FC | :: | *area* CBG | : | *area* CBF, | 12. 5. |


From

2.

From the *Vertex* of the *Hyperbola* CBF, Let there be a *Semihyperbola* as CAF; FH its Transverſe Diameter.

Then as,

| | | | | |
|---|---|---|---|---|
| HCF | : HEF | :: CBC | : ELE, | 11. Prop. |
| HCF | : HEF | :: CAC | : ENE, | 11. Prop. |
| CBC | : CAC | :: ELE | : ENE, | 11. 5. |
| CB | : CA | :: EL | : EN | 22. 6. |
| CB | : CA | :: *area* CBF | : *area* CAF, | 12. 5. |

3.

Let CVQ be ¼ of an *Ellipſis*, its Tranſverſe Diameter FV, its *ſemiconjugate* CQ, Let CGQ be ¼ of another *Ellipſis* its Tranſverſe Diameter EG, its *conjugate* CQ.

Let it be made, as,

| | | | | |
|---|---|---|---|---|
| FC | : EC | :: CV | : CG, | |
| FC | : EC | :: $\frac{CV}{2}$ | : $\frac{CG}{2}$ | 17. 7. |
| FC | : EC | :: CG | : CD, | 17. 7. |
| FC | : CG | :: EC | : CD, | 16. 5. |
| FC | : FC+CG | :: EC | : EC+CD, | 18. 5. |
| FC | : FG | :: EC | : ED, | 18. 5. |
| CV | : GV | :: CG | : DG, | 17. 7. |
| FCV | : FGV | :: ECG | : EDG, | 23. 6. |
| FCV | : FGV | :: CQC | : GLG, | 10. Prop. |
| ECG | : EDG | :: CQC | : DHD=GLG, | 10. Prop. |

F Therefore

Therefore D H and G L are equal, Because the Line C V is divided in G in the same *ratio* as C G is in D.

Therefore, as,

| | | | | |
|---|---|---|---|---|
| CV : VG | :: CG | : GD, | | |
| CV : CG | :: CG | : DC, | : | 19. 5. |
| CV : CG | :: IL | : IH, | | |
| CV : CG | :: *area* CQV | : *area* CQG, | : | |

4.

From the *vertex* of the Quarter of the *Ellipsis* C G Q ; Let there be ¼ of another *Ellipsis*, as C G R, its Transverse Diameter E G the *semiconjugate* C R,

Then, as,

| | | | |
|---|---|---|---|
| ECG : EDG | :: CQC | : DHD, | 10. Pr. |
| ECG : EDG | :: CRC | : DKD, | 10. Pr. |
| CQC : DHD | :: CRC | : DKD, | 11. 5. |
| CQ : CR | :: DH | : DK, | 22. 6. |
| CQ : CR | :: *area* CDQ | : *area* CGR, | 12. 5. |

5.

Let C G B be half a *parabola*, its Diameter C G, its Ordinate C B. Let C V B be half of another *parabola*, its Diameter C V, its Ordinate C B.

Let


Let it be,

$CV : CG :: CV : CG,$

$CV : CG :: \frac{CV}{2} : \frac{CG}{2},$

$CV : CG :: GV : GD,$

$CG : GD :: CBC : DND,$ 9. Prop.

$CV : GV :: CBC : GTG = DND,$ 9. Prop.

Therefore G T is equal to D N; because the Line C V is divided in G in the same *ratio* as C G is in D.

Therefore,

| | | | | | | |
|---|---|---|---|---|---|---|
| CV | : CG | :: | CG | : CD, | | |
| CV | : CG | :: | AT | : AN, | | |
| CV | : CG | :: | *area* CVB | : *area* CGB, | | 12. 5. |

6.

From the *vertex* of the *semiparabola* C G B; Let there be another *semiparabola*, as C G Z,

Then, as,

| | | | | | |
|---|---|---|---|---|---|
| GC | : GD | :: | CBC | : DND, | 9. Pr. |
| GC | : GD | :: | CZC | : DOD, | 9. Pr. |
| CZC | : CBC | :: | DOD | : DND, | 11. 5. |
| ZC | : BC | :: | OD | : ND, | 22. 6. |
| ZC | : BC | :: | *area* ZGC | : *area* BGC, | 12. 5. |

7.

Hence,

It follows that the *areas* of *hyperbolas*, *Ellipses* and *parabolas* may be increased or decreased in any proportion assigned; either according to their Transverse diameters or Ordinates: or both together.

It

It follows alſo,

That the *areas* of *hyperbolas*, *ellipſes* and *parabolas* of the ſame *baſes* are as their Altitudes.

And alſo,

That the *areas* of *hyperbolas*, *ellipſes* and *parabolas* of the ſame Altitudes are as their *baſes*.

Further it follows,

If there be two *hyperbolas*, namely, A and B; if the Tranſverſe diameter of A, be to the Tranſverſe diameter of B; as the conjugate diameter of A, is to the conjugate diameter of B; if the Tranſverſe diameter of A, is to the Tranſverſe diameter of B; as the interpreted diameter of A, is to the interpreted diameter of B: if the conjugate diameter of A, is to the conjugate diameter of B; as the Ordinate of A, is to the Ordinate of B. Then ſuch *hyperbolas* are called like *hyperbolas*.

And the *area* of A, will be to the *area* of B; as the Rectangled Figure of the Tranſverſe and conjugate diameters of A, to the Rectangled Figure of the Tranſverſe and conjugate diameters of B.

Or,

The *area* of A, will be to the *area* of B; as the Rectangled Figure of the interpreted diameter, and Ordinate of A, is to the Rectangled Figure of the interpreted diameter and Ordinate of B.

8.

Here note,

In the *hyperbolas* CFB, CFA and CGB the Lines CB, EL; and CA, EN; and CB, FM, are called *Ordinates*.

In the *Ellipses*,

CGQ, CGR and CVQ; the Lines DH, and DK and GL are *Ordinates*; the Lines CQ and CR conjugate diameters.

In the *parabolas*.

CGB, CGZ and CVB; the Lines CB, DN; and CZ, DO; and CB, GT are called *Ordinates*.

9.

In the *hyperbola* CFB; CF, EF, CB and EL being given, to find the Transverse dameter FH.

Let FE=A.FC=B.CB=D.EL=E.Z=HF. Therefore HE = Z + A. HC = Z + B.

Therefore,

$DD : EE :: ZB+BB : ZA+AA,$ 11. Prop.

$ZADD+\underline{AADD}=\underline{ZBEE}+BBEE,$ 16. 6.

$-ZBEE \qquad -AADD.$

*In Words thus.*

The difference between the ſquare of B in the ſquare of E, and the ſquare of A in the ſquare of D; being divided by the difference between A in the ſquare of D and B in the ſquare of E; the Quotient is the value of Z.

---

10.

In the *Ellipſis* CQG; CQ, DH, and CD being given, to find the Tranſverſe diameter EG.

Let CQ=A. DH=B. CD=D, GC=Z.
Therefore Z+D=ED. Z−D=DG.

ZZ : Z+D in Z−D :: AA : BB, 10. Prop.

---

ZZBB=ZZAA−DDAA. 16. 6.

---

DDAA=ZZAA
−ZZBB

*In Words thus.*

As the ſquare Root, of the difference between the ſquare of A and the ſquare of B, is to A, ſo is D, to Z, the *ſemitranſverſe* diameter. 22. 6.

11.

In the *parabola* CGB; CB, CD and DN being known, to find the diameter DG.

Let CB=A. DN=B. CD=C. DG=Z. Then C+Z=CG. AA : BB :: C+Z : Z, 9. Prop.

AAZ=BBC+BBZ 16. 6.

—BBZ

*In Words thus.*

As the difference betwixt the square of A and B, is to the square of B; so is C, to Z,

Or thus.

| | | | | |
|---|---|---|---|---|
| AA | : BB | :: C+Z | : Z, | 9. Prop. |
| AA—BB | : BB | :: C+Z—Z | : Z, | 17. 5. |

12.

Yet further, it may be made manifest, that by the points of B and F there may be infinite hyperbolick lines pass: the *area* of the least, shall not be so little as half the *parallelogram*, whose *base* is BC, and Altitude CE: nor the *area* of the greatest, so great, as three fourths of the said *parallelogram*.

PROP.

## PROPOSITION XXIV.

### *Of all manner of* cylindrick hoofs.

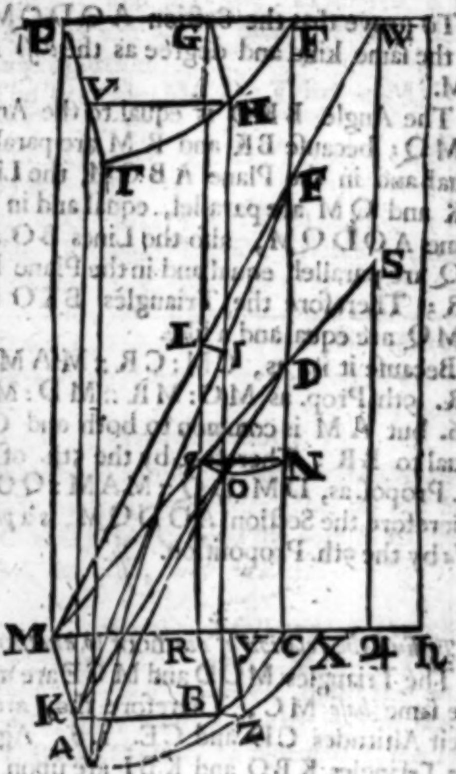
I.

LEt MACFTP be half a *parabolick Cylinder*, the *base* MAC parallel, equal and a like to the *base* FTP. Let C and F be the *vertices* of the *parabolas* ACM and TFP, CM and FP their diameters. Let this *cylinder* be cut by the Plane HVKB parallel to the Plane FPMC, the Line HB in the *superficie* of the *cylinder*. Let it be cut by another Plane, as HGRB parallel to the Plane PTAM, the line HB in the *superficie* of the said *cylinder*. Let it be cut by the Plane OADQM, the line AOD in the *superficie* of the *cylinder*, and the line DQM in the Plane CFPM; the Ordinate AM the common Section of the Planes ABCM and AODQM. Further, Let it be cut by the Plane AIELM, the line AIE in the *superficie* of the *cylinder* and the line ELM in the Plane CFPM. From K, to O and I let lines be drawn, and also from M to D and E: from the *intersection* of GR and ME, that is from L, to the *intersection* of the lines HB and AE, that is to I, let there be a line drawn, as LI; and

and likewise the line QO; then the line LI and QO will be equal and parallel to the line RB. Because the Planes AODQM and AIELM, cut the Plane GFPM at right Angles; and the points B, O, I, H, are in the *superficie* of the *cylinder* and in the line BH.

Betwixt the Planes ACM and AODQM there is a solid made, as ABCMQDOA; also betwixt the said *base* ABCM and the Plane AIELM there is another solid made as ABCMLEIA: such solids are called *cylindrick hoofs*, and they take their particular name from such *cylinders* as they are part of; *viz.* if the *base* ABCM be half, or a quarter of a circle, it may be called a circular *cylindrick hoof*; If half or a quarter of an *ellipsis*, then an *elliptick cylindrick hoof*. If the *base* be half or a whole *parabola*, then a *parabolick cylindrick hoof*. If half, or a whole *hyperbola*, then a *hyperbolick cylindrick hoof*; the like for any other.

To




To

2.

To prove that the Section AODQM is of the same kind and degree as the *base* AB CM.

The Angle BKO is equal to the Angle RMQ; because BK and RM are parallel, equal and in the Plane ABCM, the Lines OK and QM are parallel, equal and in the Plane AODQM, also the Lines BO and RQ are parallel, equal and in the Plane BH GR; Therefore the Triangles BKO and RMQ are equal and a like.

Because it is, as, CM : CR :: MAM : R BR. 9th. Prop. as MC : MR :: MD : MQ 4.6. but AM is common to both and OQ equal to BR; Therefore by the 5th. of the 23. Propos. as, DM : DQ :: MAM : QOQ. Therefore the Section AODQM is a *parabola* by the 9th. Proposition.

3.

*To find the relation of one* hoof *to another.*

The Triangles MCD and MCE are upon the same *base* MC, Therefore they are as their Altitudes CD and CE. 1.6. Again, the Triangles KBO and KBI are upon the same *base* KB, Therefore they are as their Altitudes BO and BI, 1.6.

Further,

Further, the Triangles KBO and MCD are alike, and also the Triangles KBI and MCE are alike, therefore their sides are in proportion. 4. 6.

The Triangle MCD : Triangle MCE :: CD : CE. 1. 6.

The Triangle KBO : Triangle KBI :: BO : BI. 1. 6.

But BO : BI :: CD : CE. 4. 6.

The Triangle KBO : Triangle KBI :: CD : CE. 11. 5.

KBO + MCD : KBI + MCE :: CD : CE. 12. 5.

Hence it follows,

That the solidities of *hoofs* upon the same *base* are as their Altitudes, that is, the *hoof* ABCMQDOA, is to the *hoof* ABCML EIA; as CD, is to CE.

Further it follows.

Because it is BO : BI :: CD : CE, Therefore the *superficies* of *hoofs*, upon the same *base* are as their Altitudes, that is, the *superficie* CBAOD, is to the *superficie* CBAIE; as CD, is to CE.

To

*To find the solidity of cylindrick hoofs.*

The Triangles MCD and KBO are alike. Therefore as KB : BO :: MC : CD, 4. 6. and KB : $\frac{BO}{2}$ :: MC : $\frac{CD}{2}$, by the converse of 17. 7. then as KB, to a Geometrical mean proportion between KB and half BO, so is MC, to a Geometrical mean proportion between MC and half CD, by the 22. Proposition. Let MX be made equal to the last term in the last Proportion, *viz.* the Geometrical mean proportion between MC and half CD. Then will KZ be equal to a Geometrical mean proportion between KB and half BO, and AZKM will be a *semiparabola* by the 5th of the 25 Proposition; that is, as CM : XM :: BK : ZK. Further the *area* of the Triangle MCD is equal to the Product of MC in half CD, that is, a Square whose side is a Geometrical mean proportion between MC and half CD, that is equal to the Square of MX. Also the *area* of the Triangle KBO is equal to the Product of KB in half BO, that is, equal to a square whose side is a Geometrical mean proportion between KB and half BO; that is equal to the Square of KZ.

Hence

5.

Hence it follows.

That the solidity of the *hoof* MQDOA BC is equal to all the squares in one eighth of a *parabolick Spindle* whose *semiaxis* is AM and *semidiameter* is MX; but all the squares in a *parabolick Spindle* is to a *parallelepipedon* of the same *Base* and Altitude; as 8, is to 15. *Bonav. Caval. Exerc. quer.* page 282.

6.

It further follows.

As all the squares in the whole solid AZ RM are equal to the whole *hoof* MQDO AC, so are their parts equal, if the *axis* be cut by a Plane at right Angle.

*Example.*

The Plane BKO cuts the *axis* AM in K at right Angle, that is, the Plane KBO is parallel to the Plane MCD, the part of the *Spindle* AKZ, is equal to the part of the *hoof* AKOB.

7.

If the *hoof* ABCMQDOA be cut by a Plane QON parallel to the *Base* ABCM, the part QOND may be found thus.

First, take away the part KABO equal to the part of the *Spindle* KAZ: then take away

the

the *prisme* KBRMQO; Lastly, take away the *parabolick Semicylinder* RBCNOQ, there will remain the little *hoof* QOND.

## 8.

If ACM be a *semihyperbola* its Transverse diameter C♃, the *hyperbolick cylinder* may be MABCFHTP: this *cylinder* being cut by Planes according to the *First* of this Proposition, the Sections AODQM and AI... LM will be *semihyperbolas*, DS and E... their Transverse diameters by the *First* of the 23d. Proposition. If between M♃ and half ♃S there be a Geometrical mean proportion found, suppose it M♄, and if between M... and half DG there be taken a Geometrical mean proportion, suppose it MX: And also if a Geometrical mean proportion be taken between KB and half BO, suppose it KZ; the Plane AZXM will be the *area* of a *hyperbola* its Transverse diameter X♄; by the 22d. Proposition and by the *First* of the 23d. Proposition all the Squares in one Fourth of the *hyperbolick Spindle* AZXM taking AM for its *axis*, will be equal to the *hyperbolick hoof* ABCMQDOA, by the 4th. and 5th. of this Proposition. The relations of the solidities and *superficies*, by the Third of this.

If

9.

If A B C M be a quarter of a Circle, the circular *cylinder* will be A B C M T H F P; this *cylinder* being cut by Planes, according to the *First* of this; the Sections A O D Q M and A I E L M will be quarters of *Ellipses* by the *Third* of the 23d. Proposition. If a Geometrical mean proportion be taken between M C and half C D, suppose it M X, and also a Geometrical mean proportion be taken between K B and half B O, suppose it K Z. The *area* A Z X M will be one quarter of an *Ellipsis* by the 22d. Proposition, and by the Third of the 23d. Proposition.

All the squares in one fourth of the *semispheroide*, taking A M for its *Semiaxis*, and M X for its *semidiameter*, will be equal to the circular *cylindrick hoof* A O D Q M C B A, by the Fourth and Fifth of this Proposition. As the whole, so the parts, according to the Sixth and Seventh of this Proposition. How to find all the squares in any Sphere or *spheroide* is taught Proposition the 15th. and 16th.

Here Note,

If the Altitude of the *hoof* be equal to four diameters of the *base*, that *hoof* will be equal to all the squares in a Sphere adscribed in that *cylinder*.

If the Altitude be less than Four diameter of the *base*, that *hoof* will be equal to all the squares in a *spheroide* whose longest diameter shall be the *axis*.

But if the Altitude be greater than Four diameters of the *base*, then that *hoof* will be equal to all the squares in a *spheroide* whose shortest diameter shall be the *axis*.

10.

If A B C M be a quarter of an *Ellipsis*, the *Elliptick cylinder* will be A B C M T H F P this *cylinder* being cut by *Planes*, according to the First of this, the Sections A O D Q M and A I E L M will be quarters of *Ellipse* by the 3d. of the 23d. Proposition.

If Geometrical means be taken betwee M C and half C D, and also between K and half B O, suppose them to be M X and K Z, Then will A Z X M be a quarter of a *Ellipsis* by the 22d. Proposition, and by the 3d. of the 23d. Prop. all the squares in one Fourth of a *semispheroide* taking A M for its *semiaxis* and M X for its *semidiameter*, will be equal to the *Elliptick cylindrick hoof* A O D Q M C B A by the Fourth and Fifth of this Proposition. The parts K A B O B C R Q O N and Q O N D are found as in the Sixth and Seventh of this.

# A Table of Squares and Cubes.

| Roots. | Squares. | Cubes. | Roots. | Squares. | Cubes. |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 25 | 625 | 15625 |
| 2 | 4 | 8 | 26 | 675 | 17576 |
| 3 | 9 | 27 | 27 | 729 | 19683 |
| 4 | 16 | 64 | 28 | 784 | 21952 |
| 5 | 25 | 125 | 29 | 841 | 24389 |
| 6 | 36 | 216 | 30 | 900 | 27000 |
| 7 | 49 | 343 | 31 | 961 | 29791 |
| 8 | 64 | 512 | 32 | 1024 | 32768 |
| 9 | 81 | 729 | 33 | 1089 | 35937 |
| 10 | 100 | 1000 | 34 | 1156 | 39304 |
| 11 | 121 | 1331 | 35 | 1225 | 42875 |
| 12 | 144 | 1728 | 36 | 1296 | 46656 |
| 13 | 169 | 2197 | 37 | 1369 | 50653 |
| 14 | 196 | 2744 | 38 | 1444 | 54872 |
| 15 | 225 | 3375 | 39 | 1521 | 59319 |
| 16 | 256 | 4096 | 40 | 1600 | 64000 |
| 17 | 289 | 4913 | 41 | 1681 | 68921 |
| 18 | 324 | 5832 | 42 | 1764 | 74088 |
| 19 | 361 | 6859 | 43 | 1849 | 79507 |
| 20 | 400 | 8000 | 44 | 1936 | 85184 |
| 21 | 441 | 9261 | 45 | 2025 | 91125 |
| 22 | 484 | 10648 | 46 | 2116 | 97336 |
| 23 | 529 | 12167 | 47 | 2209 | 103823 |
| 24 | 576 | 13824 | 48 | 2304 | 110592 |

# *A Table of Squares and Cubes.*

| Roots. | Squares. | Cubes. |
|---|---|---|
| 49 | 2401 | 117649 |
| 50 | 2500 | 125000 |
| 51 | 2601 | 132651 |
| 52 | 2704 | 140608 |
| 53 | 2809 | 148877 |
| 54 | 2916 | 157464 |
| 55 | 3025 | 166375 |
| 56 | 3136 | 175616 |
| 57 | 3249 | 185193 |
| 58 | 3364 | 195112 |
| 59 | 3481 | 205379 |
| 60 | 3600 | 216000 |
| 61 | 3721 | 226981 |
| 62 | 3844 | 238328 |
| 63 | 3969 | 250047 |
| 64 | 4096 | 262144 |
| 65 | 4225 | 274625 |
| 66 | 4356 | 287496 |
| 67 | 4489 | 300763 |
| 68 | 4624 | 314432 |
| 69 | 4761 | 328509 |
| 70 | 4900 | 343000 |
| 71 | 5041 | 357911 |
| 72 | 5184 | 373248 |
| 73 | 5329 | 389017 |
| 74 | 5476 | 405224 |
| 75 | 5625 | 421875 |
| 76 | 5776 | 438976 |
| 77 | 5929 | 456533 |
| 78 | 6084 | 474552 |
| 79 | 6241 | 493039 |
| 80 | 6400 | 512000 |
| 81 | 6561 | 531441 |
| 82 | 6724 | 551368 |
| 83 | 6889 | 571787 |
| 84 | 7056 | 592704 |
| 85 | 7225 | 614125 |
| 86 | 7396 | 636056 |
| 87 | 7569 | 658503 |
| 88 | 7744 | 681472 |
| 89 | 7921 | 704969 |
| 90 | 8100 | 729000 |
| 91 | 8281 | 753571 |
| 92 | 8464 | 778688 |
| 93 | 8649 | 804357 |
| 94 | 8836 | 830584 |
| 95 | 9025 | 857375 |
| 96 | 9216 | 884736 |

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## A Table of Squares and Cubes.

| Roots. | Squares. | Cubes. | Roots. | Squares. | Cubes. |
|---|---|---|---|---|---|
| 97 | 9409 | 912673 | 121 | 14641 | 1771561 |
| 98 | 9604 | 941192 | 122 | 14884 | 1815848 |
| 99 | 9801 | 970299 | 123 | 15129 | 1860867 |
| 100 | 10000 | 1000000 | 124 | 15376 | 1906624 |
| 101 | 10201 | 1030301 | 125 | 15625 | 1953125 |
| 102 | 10404 | 1061208 | 126 | 15876 | 2000376 |
| 103 | 10609 | 1092727 | 127 | 16129 | 2048383 |
| 104 | 10816 | 1124864 | 128 | 16384 | 2097152 |
| 105 | 11025 | 1157625 | 129 | 16641 | 2146689 |
| 106 | 11236 | 1191016 | 130 | 16900 | 2197000 |
| 107 | 11449 | 1225043 | 131 | 17161 | 2248091 |
| 108 | 11664 | 1259712 | 132 | 17424 | 2299968 |
| 109 | 11881 | 1295029 | 133 | 17689 | 2352637 |
| 110 | 12100 | 1331000 | 134 | 17956 | 2406104 |
| 111 | 12321 | 1367631 | 135 | 18225 | 2460375 |
| 112 | 12544 | 1404928 | 136 | 18496 | 2515456 |
| 113 | 12769 | 1442897 | 137 | 18769 | 2571353 |
| 114 | 12996 | 1481544 | 138 | 19044 | 2628072 |
| 115 | 13225 | 1520875 | 139 | 19321 | 2685619 |
| 116 | 13456 | 1560896 | 140 | 19600 | 2744000 |
| 117 | 13685 | 1601613 | 141 | 19881 | 2803221 |
| 118 | 13924 | 1643032 | 142 | 20164 | 2863288 |
| 119 | 14161 | 1685159 | 143 | 20449 | 2924207 |
| 120 | 14400 | 1728000 | 144 | 20736 | 2985984 |

# A Table of Squares and Cubes.

| Roots. | Squares. | Cubes. |
|---|---|---|
| 145 | 21025 | 3048625 |
| 146 | 21316 | 3112136 |
| 147 | 21609 | 3176523 |
| 148 | 21904 | 3241792 |
| 149 | 22201 | 3307949 |
| 150 | 22500 | 3375000 |
| 151 | 22801 | 3442951 |
| 152 | 23104 | 3511808 |
| 153 | 23409 | 3581577 |
| 154 | 23716 | 3652264 |
| 155 | 24025 | 3723875 |
| 156 | 24336 | 3796416 |
| 157 | 24649 | 3869893 |
| 158 | 24964 | 3944312 |
| 159 | 25281 | 4019679 |
| 160 | 25600 | 4096000 |
| 161 | 25921 | 4173241 |
| 161 | 26244 | 4251528 |
| 163 | 26569 | 4330747 |
| 164 | 26896 | 4410944 |
| 165 | 27225 | 4492125 |
| 166 | 27556 | 4574296 |
| 167 | 27889 | 4657463 |
| 168 | 28224 | 4741632 |

| Roots. | Squares. | Cubes. |
|---|---|---|
| 169 | 28561 | 4826809 |
| 170 | 28900 | 4913000 |
| 171 | 29241 | 5000211 |
| 172 | 29584 | 5088448 |
| 173 | 29929 | 5177717 |
| 174 | 30276 | 5268024 |
| 175 | 30625 | 5369375 |
| 176 | 30976 | 5451776 |
| 177 | 31329 | 5545233 |
| 178 | 31684 | 5639752 |
| 179 | 32041 | 5735339 |
| 180 | 32400 | 5832000 |
| 181 | 32761 | 5929741 |
| 182 | 33124 | 6028568 |
| 183 | 33489 | 6128487 |
| 184 | 33856 | 6229504 |
| 185 | 34225 | 6331625 |
| 186 | 34596 | 6434856 |
| 187 | 34969 | 6539203 |
| 188 | 35344 | 6644672 |
| 189 | 35721 | 6751269 |
| 190 | 36100 | 6859000 |
| 191 | 36481 | 6967871 |
| 192 | 36864 | 7077888 |

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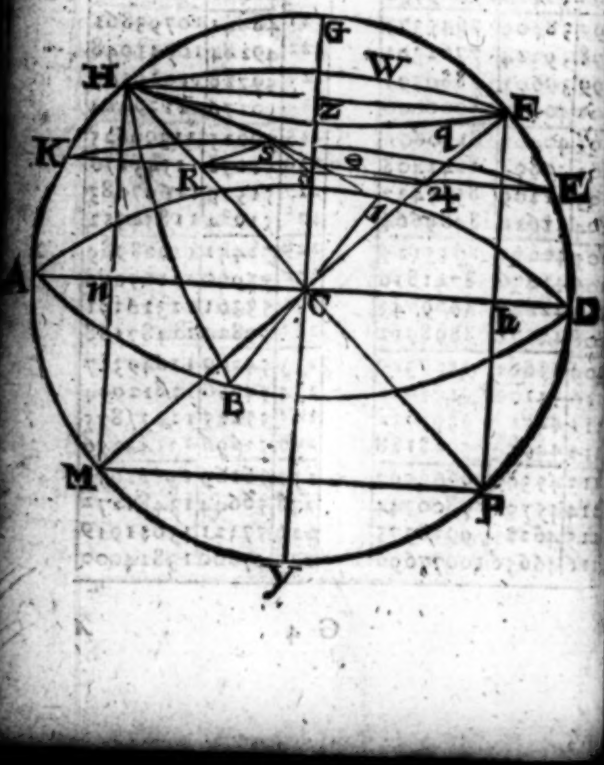
## A Table of Squares and Cubes.

| Roots. | Squares. | Cubes. | Roots. | Squares. | Cubes. |
|---|---|---|---|---|---|
| 193 | 37249 | 7189057 | 217 | 47089 | 10218313 |
| 194 | 37636 | 7301384 | 218 | 47524 | 10360232 |
| 195 | 38025 | 7414875 | 219 | 47961 | 10503459 |
| 196 | 38416 | 7529536 | 220 | 48400 | 10648000 |
| 197 | 38809 | 7645373 | 221 | 48841 | 10793861 |
| 198 | 39204 | 7762392 | 222 | 49284 | 10941048 |
| 199 | 39601 | 7880599 | 223 | 49729 | 11089567 |
| 200 | 40000 | 8000000 | 224 | 50176 | 11239424 |
| 201 | 40401 | 8120601 | 225 | 50625 | 11390625 |
| 202 | 40804 | 8242408 | 226 | 51076 | 11543176 |
| 203 | 41209 | 8365427 | 227 | 51529 | 11697083 |
| 204 | 41616 | 8489664 | 228 | 51984 | 11852352 |
| 205 | 42025 | 8615125 | 229 | 52441 | 12008989 |
| 206 | 42436 | 8741816 | 230 | 52900 | 12167000 |
| 207 | 42849 | 8869743 | 231 | 53361 | 12326391 |
| 208 | 43264 | 8998912 | 232 | 53824 | 12487168 |
| 209 | 43681 | 9129329 | 233 | 54289 | 12649337 |
| 210 | 44100 | 9261000 | 234 | 54756 | 12812904 |
| 211 | 44521 | 9393931 | 235 | 55225 | 12977875 |
| 212 | 44944 | 9528128 | 236 | 55696 | 13144256 |
| 213 | 45369 | 9663597 | 237 | 56169 | 13312053 |
| 214 | 45796 | 9800344 | 238 | 56644 | 13481272 |
| 215 | 46225 | 9938375 | 229 | 57121 | 13651919 |
| 216 | 46656 | 10077696 | 240 | 57600 | 13824000 |

## PROPOSITION XXV.

LEt AYDG be a *Sphere*, GY its *axis*, C the Center of the *Sphere*, the Lines AD and BI be at right Angles, and at right Angles with the *axis* GY.



Let this *Sphere* be cut by a *Plane* through the *axis* G Y, and the *Base* A D; the Section makes the Circle A Y D G.

Let the *Sphere* be cut by another *Plane*, viz. as by the *Plane* C B H S I; H C its diameter, B I its diameter of the *Base*. In the inclining Solid whose diameter of the *Base* is B I, and diameter of the Section C H and the altitude C Z or H N, that is the inclining *Solid* B H S I C is equal to the Zone A H F D, less the *Cone* H W F Q C, or the inclining *Solid* B H S I C is equal to the *Excavatus* part of the *sphere* A C H F C D. Let the *plane* K S E be parallel to the *plane* A I D; R S the common section of the *planes* K S E and C H I: O E, O S and O K are equal; the square of O S is equal to the squares of R O and R S. 47. 1. Therefore the *Area* of the Circle who semidiameter is O S, is equal to the *Areas* of the two Circles whose semidiameters are O R and R S, that is, the *Area* of the semicircle O K S E, less the *Area* of the semicircle O R ⊖ ♃ is equal to the *armille* ♃ ⊖ R K S E, that is, the *Area* of a semicircle whose semidiameter is R S. Wherever the *plane* K S E be drawn parallel between the two parallel *planes* A I D and H W F it will have the same qualification by the 47. 1.

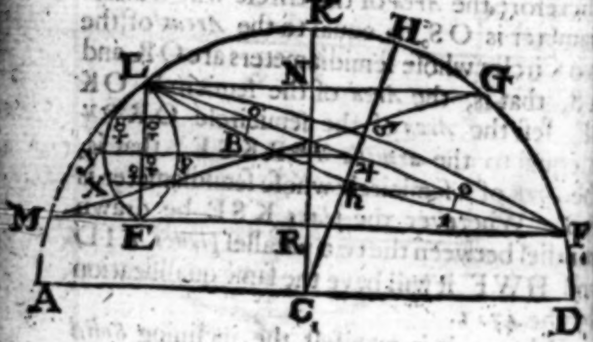
Whence it is manifest the inclining *Solid* B H S I C is the difference betwixt the *Zone*

A

AHFD. and the *Cone* HWFQC. Further this inclining *Solid* BHSIC is equal to a *hemisspheroide* whose *axis* is HN, the altitude of the section BHSIC, and the semidiameter of the *Base* of the *hemisspheroide* is CA. By the 10th. of the 6th. of *Euclid*, and by the 3d. of the 23. Proposition.

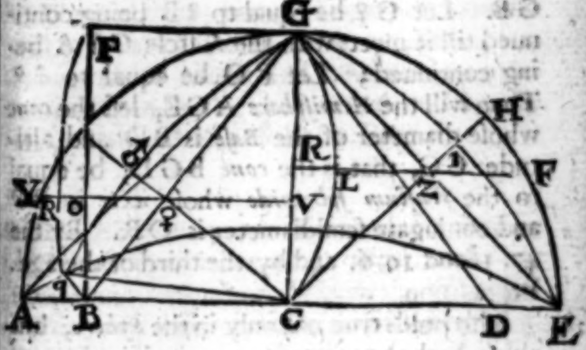
2.

Let AKD be a *Hemisphere*; the *Plane* AKD and the *Plane* L♄F♂ cutting one another at right Angles their common section the line LF. From the points L and F draw the Lines as LG and FM parallel to the line AD, also from the points L and G, to the points F




and

…nd M let there be lines drawn as G M and … F their common section the point Z. From … the point L, to the line M F let there be a perpendicular line as LE. Let YP be equal to … then will the *Spheroide* whose *axis* is E L and its conjugate diameter Y P be equal to the *Excavatus* part MZLGZF of the Zone MLGF; or the Zone MLGF less the *cones* GZL and FZM will be equal to the *Spheroide* EYLP. By the 47. of the 1. of *Euclid*, and by the 10th. 6th. of *Euclid*, and by the third. of the 23d. Propos. Here note, Z is the *vertex* of the two *cones*. Here also note, that … L ♃ is equal to ♃ F. Further note, the lines … IO, ♄ ♃ and BO are supposed to be at right Angles with the line L F.



3.

Let AGE be a *Hemisphere*; the *planes* AGE and GIE cutting one another at right Angles; their common section the line GE. Let GZ be equal to ZE; and also RL equal to ZI; then will the *Hemisphere* AGE less the *cone* whose diameter of the *Base* is AE and Altitude CG, that is the *Cone* AGE, be equal to a *Spheroide* whose *axis* is CG and conjugate semidiameter is RL. By the 47th. 1. and 10th. and 6th. and by the third of the 23d. Prop.

4.

Let AGE be a *Hemisphere*; the *Planes* AGE and G♂Q cutting one another at right Angles; their common section the line GB. Let G♀ be equal to ♀B being continued till it meets with the Circle GYA being continued; Let RO be equal to ♂♀. Then will the *Hemisphere* AGE, less the *cone* whose diameter of the *Base* is BD and altitude CG, that is the *cone* BGD, be equal to the *frustum spheroide* whose *axis* is BP and conjugate semidiameter is OR. By the 47. 1. and 10. 6. and by the third of the 23d. Proposition.

This holds true not only in the *Sphere*, but also in both the *Spheroides*; as well in the *cone* as also in both the *Conoides*, not only when the

the cutting *plane* cuts the *axis*, but when it is parallel thereunto; but when it is parallel to the *axis*, there will be a *cylinder* instead of these *cones*. The *Excavatus* parts in the *sphere* and both the *spheroides*, will always be *spheres* or *spheroides*, or parts thereof their demonstrations by the 47. 1. 10. 6. and by the third of the 23d. Prop.

5.

The *Excavatus* parts of a *frustum cone*, will have relation to the *hyperbola*, *ellipsis* and *parabola*; Their demonstrations from the 47. 1. 10. 6. and from the first, third and fifth of the 23. Prop.

6.

The *Excavatus* parts of a *hyperbolick conoide* will have relation to all the three sections, *viz.* the *hyperbola*, *ellipsis* and *parabola*, their demonstrations by the 47. 1. 10. 6. and by the first, third and fifth of the 23. Prop.

7.

The *Excavatus* parts of a *parabolick conoide*, will have relation but to the *ellipsis* and *parabola*, their demonstrations from the 47. 1. and 10. 6. and from the third and fifth of the 23. Proposition.

8.

The solidity of every *frustum cone*, is equal to a *cylinder* whose *Base* is the lesser *Base* of the *frustum*, and its altitude the altitude of the *frustum*, more a *hyperbolick conoide* whose *Base* is equal to the difference of the *Bases* of the said *frustum*, and its altitude the altitude of the *frustum*; the Transverse Diameter of the *hyperbolick conoide* will be a line intercepted, betwixt the continuation of the other side of the *frustum cone*, and the intercepted Diameter, being continued.

9.

The solidity of every *frustum parabolick conoide*, is equal to a *cylinder* whose *Base* is the lesser *Base* of the *frustum*, and altitude the altitude of the said *frustum*, more a *parabolick conoide* whose *Base* is equal to the difference betwixt the *Bases* of the said *frustum*, and its altitude the altitude of the *frustum*.

10.

The solidity of every *frustum hyperbolick conoide*, is equal to a *cylinder* whose *Base* is the lesser *Base* of the *frustum*, and altitude the altitude of the said *frustum*; more a *hyperbolick conoide* whose *Base* is equal to the difference betwixt the *Bases* of the said *frustum*; and

and its altitude the altitude of the *frustum*.

This latter *hyperbolick conoide*, is like to that *hyperbolick conoide*, of which the *frustum* is a part.

11.

A *Sphere* being cut by two parallel *planes*, both of them equidistant & parallel to a *plane* passing through the Center of the *Sphere*; includes a part of the *Sphere*, which for distinction sake may be called a *middle Zone*.

The solidity of every *middle Zone* of any *Sphere*, is equal to a *cylinder* of the same *Base* and altitude as the *Zone*; more a *Sphere* whose diameter is equal to the altitude of the *Zone*.

12.

A *Spheroide* being cut by two parallel *planes*, both of them equidistant and parallel to a *plane* passing through the Center of the *Spheroide* and cutting the *Axis* of the said *spheroide* at right Angles, includes a part of the *spheroide*, which part for distinction sake may be called the *middle Zone* of a *spheroide*.

The solidity of the *middle Zone* of any *spheroide*, is equal to a *cylinder* of the same *Base*, and altitude as the *Zone*; more a *spheroide* whose *Axis* is equal to the altitude of the *Zone*.

The

The *ellipsis* which generates this last *spheroide* is like to that *ellipsis* which generates that *spheroide* of which this *middle Zone* is part.

13.

The solidities of *hyperbolick conoides* upon the same *Base*, are as their altitudes. Here the Transverse Diameter is increased or decreased in the same proportion as the intercepted Diameter. By the first of the 23d Proposition.

And also,

The solidities of *hyperbolick conoides* under the same altitude, are as their *Bases*. Here the conjugate Diameter is increased or decreased in the same proportion as the ordinate. By the second of the 23d. Proposition.

The solidities of like *hyperbolick Conoides*, are in a triplicate *ratio* of their corresponding terms, that is, their Transverse Diameters, or their intercepted Diameters, their conjugate Diameters, or the Ordinates.

14.

The solidities of *hemispheroides* upon the same *Base*, are as their Altitudes. By the 3d of the 23d. Prop.

The solidities of *hemispheroides* under the same Altitude, are as their *Bases*, by the 4th of the 23d. Prop.

The

The solidities of like *spheroides*, are in a *triplicate ratio* of their corresponding terms.

15.

The solidities of *parabolick conoides* upon the same *Base*, are as their Altitudes. By the 5th. of the 23d. Prop.

The solidity of *parabolick conoides* under the same Altitude, are as their *Bases*. By the 6th. of the 23d. Prop.

The solidities of like *parabolick conoides*, are in the *triplicate ratio* of their correspon-ding terms.

*An Example thus.*

Suppose there be two *parabolick conoides*, A and B, the solidity of A, will be to the solidity of B; as the Cube of the *axis* of A, is to the Cube of the *axis* of B.

Further, as the *parabolick conoide* of A, is to the *parabolick conoide* of B; so is the Cube of *latus rectum* of A, to the Cube of *latus rectum* of B.

Yet further, if these *conoides* both equally incline, it will be;

As the *conoide* of A, is to the *conoide* of B; so will the Cube of the Diameter of A, be to the Cube of the Diameter of B. The like in the rest.

If there be two *parabolick conoides* unlike, suppose A and D. Then,

A will have that proportion to D, as is

composed of the *Base* and altitude of A, to the *Base* and altitude of D.

In *parabolick conoides*, as A and B.

If the *conoide* of A be equal to the *conoide* of B, then their *Bases* and altitudes are reciprocal, and if their *Bases* and altitudes are reciprocal; those *conoides* are equal.

Thus.

As the *Base* of A, is to the *Base* of B; so the altitude of B to the altitude of A.

These are said to be reciprocal, and their magnitudes are equal.

The like in *hemispheroides*, but not in *hyperbolick conoides*; for there may be infinite *hyperbolick conoides*, yet having the same *Base* & altitude; the least not so little as a *Cone*, nor the greatest so great as a *parabolick conoide* of that same *Base* and altitude. In this condition the semidiameter of the *Base*, and the altitude are supposed to be equal.

16.

Spheres of equal Diameters may be added together, their sum will be a *spheroide*, whose *axis* will be equal to the Diameter of one of the *spheres*; But the *area* of that Circle which passeth through the Center of this *spheroide*, and cutteth the *axis* at right Angles; will be equal to the *areas* of so many great Circles of those *spheres*, as there are *spheres* in number. Suppose there be six *spheres* to be added together,

ther, the *axis* of the *spheroide* will be equal to one of their Diameters; but the *area* of that Circle, that passeth through the Center of that *spheroide*, and cutteth the *axis* at right Angles, shall be equal to the *areas* of six Circles whose Diameter is equal to the Diameter of one of the *spheres*, The *areas* of the Circles are added, or substracted, by the 47th. of the 1. of *Euclid*.

*Spheres* and *Spheroides* of the same *axis*, may be added to, or substracted from each other, their sum, and difference will be *spheres* or *spheroides*.

17.

*Hyperbolick conoides* of the same *axis* may be added too, or substracted from each other, their sum and difference will be *hyperbolick conoides*. Here you are to take the sum of the *bases*, for the *base* of the sum, and the difference of the *bases*, for the *base* of the difference. Further, We are to take both the *parameters* for the *parameter* of the sum, and the difference of both the *parameters* for the *parameter* of the difference.

18.

*Parabolick conoides* of the same *axis*, may be added too, or substracted from each other, the sum and difference will be *parabolick conoides*, using the former rules for their *bases* and *parameters*.

Here note, the line P Z in the Diagram for the 9th. Propos. the line P N in the Diagram for the 10th. Propos. the line N H in the Diagram for the 11th. Propos. are called *parameters*.

19.

If the *axis*, Transverse and conjugate Diameters of a *hyperbolick conoide* be equal one to another; and the *axis* equal to the *axis* of a *Sphere*: this *hyperbolick conoide* and *Sphere* being added together, they make a *parabolick conoide*; its *Base* and altitude equal to the *Base* and altitude of the *hyperbolick conoide*, its *parameter* the double of the Diameter of the *Sphere*.

If there be a *hyperbolick conoide*, A; and a *Spheroide*, B; their *axis* equal: If the transverse Diameter of A, is to the *parameter* of B; as the *parameter* of A, is to the *parameter* of B. Two such *conoides* being added together, they will make a *parabolick conoide*; its *Base* the same as the *hyperbolick conoide*; its *parameter* equal to the *parmeters* of the other *conoides*, being added together.

If there be a *Cone*, whose *axis* is equal to the Diameter of the *Base*; and a *Sphere* whose *axis* is equal to the *axis* of the *Cone*; this *Cone* and *Sphere* being put together, makes a *parabolick conoide*; its *Base* equal to the *Base* of

of the *Cone*, its *parameter* equal to the Diameter of the *Sphere.*

20.

*Parabolick Conoides*, may always be added to, and sometimes substracted from *hyperbolick conoides* of the same *axis*; that sum will be a *perbolick conoide*, and that difference when a difference may be, will be a *hyperbolick conoide*, the sum of their *Bases* for the *Base* of the sum: and the difference of their *Bases* for the *Base* of their difference: the sum of their *parameters* for the *parameter* of the sum, and the difference of the *parameters* for the *parameter* of the difference.

Here note, when the *parameter* of the *hyperbolick conoide* is greater than the *parameter* of the *parabolick conoide* then a difference may be: but if it be lesser, then no difference can be taken.

The Demonstrations of these are in Prop. 15, 16, 17 and 18, compared with Propos. 9, 10 and 11. of this Book.

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APPLI-

## The Application.

THe first Proposition is to find the solidity of *pyramides* and *Cones*, or *frustum pyramides* and *Cones*, and may be applicable to the measuring of all solids or Vessels in that form, whether whole or in part, or gradually, that is, foot by foot, or inch by inch.

The second Proposition may be applyed to the measuring of irregular solids, and may be useful for the exact measuring of all sorts of Stone and Timber: also for the exact measuring of all sorts of *elliptick*, *parabolick* and *hyperbolick* irregular solids, or Vessels that are made in that form: for such solids may be cut into *parallelepipedons*, *prismes* and *pyramides*, and then reduced to their own nature, by the proportion of the *parallelogram* adscribed about those Figures to the Figures themselves, Thus.

The proportion of *parallelograms* of the same *Base* and altitude with the *areas* of *parabolas* are as 4 to 3, Therefore,

As

As 4, is to 3; so is any such solid to any such *parabolick* irregular solid.

By the help of a Table of *Squares* and *Cubes* any such solids may be calculated foot by foot, or inch by inch, without any great trouble, as is shewed in the third case of the third Proposition; This second and third Propos. are the general use in such kind of solids.

In the fourth Propos. with its several cases, there is the measuring of *frustum pyramides* when their *Bases* are not parallel.

In the fifth Propos. there is the relation of the *Sphere* and *Spheroide* to the *cylinders* of their *bases* and altitudes, as well of the parts as the whole.

In the sixth Propos. there is the measuring of the *middle Zone* of a *Sphere* and *Spheroide*; the *middle Zone* of a *Spheroide*, hath been taken generally for the Figure representing a Cask; so, the measuring of one, the other is measured.

In the twelfth Propos. there is the measuring of a portion of a *Sphere*; which may be applyed to the measuring of the inverted Crown of Brewers Coppers, or several other uses.

In the thirteenth Propos. there is the measuring of *parabolick conoides*, which may be taken for a Brewers Copper, the inverted crown.

Sometimes it may be a portion of a *Sphere*, or *Spheroide*, but sometimes the portion of a *parabolick conoide*; other times the portion of a *hyperbolick conoide*, they ought to be taken as discretion seems convenient.

In the fourteenth Propos. there is the measuring of a *hyperbolick conoide*, which may be token for a Brewers Copper.

In Propos. 15, 16, 17, and 18. there is the measuring of a *sphere*, *spheroide*, *parabolick conoide* and *hyperbolick conoide*, as well the whole as their parts.

If the *parabolick* or *hyperbolick conoides* be taken for Brewers Coppers, with the help of the Tables of Squares and Cubes, they may easily be calculated foot by foot, or inch by inch according to the third Prop.

In the twentyeth Prop. there is the masuring of Circular and *Elliptick splindles*.

The *middle Zone* may be taken for a Cask.

In the twenty first Propos. there is the measuring of the second Section in a *sphere* and *spheroide*.

The use may be to measure the *middle Zone* of a *spheroide*, cut by a *plane* parallel to the *axis*; that is, when the *superfice* of the liquor cuts the heads of the Cask.

In the twenty fourth Propos. there is the meesuring of right *cylindrick hoofs*, *viz. Circular*, *Elliptick*, *parabolick* and *hyperbolick*; and

and may be used for the measuring of Brewers leaning Vessels.

If a Brewers Copper be taken to be of that Figure that *parabolick* or *hyperbolick conoides* are, and they stand leaning, the measuring of them is almost the same as though they did not lean.

Here I ought to have shewed the making, and use of an Instrument for taking the leaning of such Vessels; But my business calls me off; However, they may be had of Mr. *John Marks* Instrument maker, living at the Sign of the *Ball* near *Somerset House* in the *Strand*, who was formerly Servant to that incomparable Instrument maker Mr. *Henry Sutton*.

Here note, the Table of Squares and Cubes is very ready and useful in finding the portions of a *sphere*, *spheroide*, *parabolick* and *hyperbolick conoides*.

*To find two such numbers, that their Product being added to the sum of their squares, the sum shall be a square, and its Root commensurable.*

LEt one of the numbers be A, and the other Z. The product may be A Z; the sum of their squares Z Z + A A; the sum of their squares and product may be Z Z + Z A + A A, equal to a square whose side is, Z + 2 that

that is the ſquare $ZZ - 4Z + 4$, therefore $ZZ + ZA + AA = ZZ - 4Z + 4$, that is, $ZA + AA = -4Z + 4$; Let A be a unit, then $5Z = 3$, that is, $Z = \frac{3}{5}$, that is, Z is 3, and A, 5. Theſe two numbers make good the queſtion, for 3 in 5, is 15; the ſquare of 3, 9, and the ſquare of 5, is 25; their ſum is 49, whoſe Root is 7. *Albert Girad* obſerves from this ſeventh Propoſ. of the 5th. of *Diophantus*, That if there be a Triangle made of 3 ſuch numbers, the Angle oppoſite to the greateſt ſide will be 120 degrees. It may be further obſerved, that if there be 2 right angled Triangles made of theſe 3 numbers, the ſum of the *hypotenuſe* and *baſe* of the one, will be equal to the ſum of the *hypotenuſe* and *baſe* of the other; and alſo the *area* of the one ſhall be equal to the *area* of the other.

Thus,

| | | | | |
|---|---|---|---|---|
| 7 | 49 | 7 | 49 | |
| 5 | 25 | 3 | 9 | |
| | 74 | | 58 | The ſum of their Squares. |
| | 24 | | 40 | The difference of their Squares. |
| | 70 | | 42 | Their double Rectangles. |
| | 98 | | 89 | The ſum of their ſides. |
| | 840 | | 840 | Their *Areas*. |

Here note, the double Rectangles are taken for their Altitudes.

Here

Here note,

In Progressions from a Unit, the sum, of the sum and difference of the greatest number in that progression, and any one number betwixt the greatest and Unity, is equal to the sum, of the sum and difference of that same greatest number; and any other number betwixt Unity and that greatest.

Further note,

This seventh Proposition is a *Lemma* to the eighth, to find three Triangles of equal *areas*; therefore the *areas* are equal, and the *hypotenuse* and *base* of the one, is equal to the *hypotenuse* and *Base* of the other.

A general Theorem for the finding of two such numbers; Take the square of any number, from which take a Unit; take the double of that number, to which adde a Unit; that sum and difference will be the two numbers required. Thus,

The number taken is 3, its square 9, less a unit is 8; the double of 3, is 6 more, a unit is 7; these two numbers makes good the question.

For $56+64+49=169$, whose root is 13.

Two right angled Triangles being made of these three numbers, *viz.* 7, 8, 13. according to the former method, will have that same qualification.

This

This Proposition was publickly proposed [illegible] PARIS in the year 1633, which *Renatos d[illegible] Cartes* resolved, and *Francis Schooten* publi[illegible]ed in Sect. 12. *Miscel.* incumbred with a squa[illegible] adfected equation, with *surds*.

*F I N I S.*

*Errata.*

Page 17, line 3, for *Ecliptick* read *Elliptick*. p. 25, for *Exca[illegible]tum* r. *Excavatum*. p. 29, l. 12, for $\frac{1}{3}$, r. $\frac{1}{6}$. In the same line, for B[illegible] BFE; r. BCBFD. p. 30, l. 22, for ZB, r. ZC. p. 32, l. 1[illegible] for NR, r. NP. p. 39, l. 13, put ; after Line. p. 46, l. 2[illegible] for 634, r. 624. p. 53, l. 3, for APR, r. APK. p. 48, l. 2[illegible] for ZB, r. XB. p. 60, l. *ult.* for R ⊖♀♃, r. R ⊖♂♃. p. 61, l. 6, for V, r. VI. p. 64, l. 4, for GG, r. GC. p. 6[illegible] l. 18, for CDQ, r. CGQ. p. 70, p. 71 l. 5, put , after D[illegible] p. 74, l. 4, for GFPM; r. CFPM. p. 80, l. 21, put : befo[illegible] all. p. 82, l. 25, after KABQ put ,

*Imprimatur.*

*Tho. Tomkins* R. Rmo in Christo Patri ac Domino Do[illegible] *GILBERTO* Divinâ Providantiâ Archiep[illegible] *Cant.* à Sac. Dom.

Ex Æd. *Lamb.*
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